

Holes in a polytope defined by degree sequences

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Abstract

For a family K of distinct 3-element subsets (a 3-family) of $[n] := \{1, 2, \dots, n\}$, define $d(K) = (d_1(K), d_2(K), \dots, d_n(K))$, where $d_i(K) := |\{S \in K : i \in S\}|$ for $i \in [n]$. Let D_n be the convex hull of the set of vectors $d(K)$ for 3-families. A well - known open problem is to determine if D_n contains any points, with integer coordinates that sum to a multiple of 3, that are not of the form $d(K)$ for any 3-family.

The answer to the analogous problem for 2-families (graphs) is known. I will sketch the theory for 2-families and describe recent progress by Klivans and Reiner for 3-families.

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