An Attempt to Turn Geometry into Decorated Graphs

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Abstract

In the late 19th century, mathematicians were interested in problems such as this one: given four generically placed lines in three dimensions, how many other lines intersect all four? This question and many others can be formulated in terms of the intersections of subvarieties of the Grassmannian of k-planes in n-space, or more generally, flag varieties (whose points are sequences of inclusions of vector spaces).

These intersection questions inside the flag variety and some generalizations, together with related algebraic and combinatorial questions, form the field of Schubert calculus. Of primary importance is that flag varieties can be realized as algebraic, symplectic manifolds with Hamiltonian actions by a compact torus. Among the magic properties are that the torus acts with isolated fixed points, and that codimension-one tori fix only points and two-spheres.

The desire to compute associated algebraic invariants, such as the product structure of associated rings in special bases, has spawned many combinatorial and graph-theoretic objects. In this talk, we will discuss some graphs associated to certain manifolds with torus actions, and ask the question of how combinatorial games involving the graphs can be used to answer geometric questions about the original manifold and intersections of subvarieties therein.

Keywords: flag variety, Schubert calculus