

Combinatorial Hopf algebras by way of groups

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Abstract

Hopf algebras are a natural algebraic tool for studying combinatorics, but their definition is often seen as complicated and esoteric. In this talk we introduce Hopf algebras by way of a related and more familiar algebraic concept: groups. We show how both structures can be defined similarly in terms of commutative diagrams, but Hopf algebras differ in that they have a more complicated “coproduct.” The coproduct is a map with one input and a pair of outputs. In combinatorics, this map is used to encode certain decomposition laws. For example, a graph can be decomposed into complementary pairs of subgraphs and a word can be deconcatenated into complementary subwords. We will give a number of examples from combinatorics and discuss some related constructions.

Keywords: Hopf algebras