# 4.3. Differentiation of Logarithmic and Exponential Functions

Derivative of ln x

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \text{for} \quad x > 0$$

#### Example

Differentiate the function  $f(x) = x \ln \sqrt{x}$ .

## **Differentiation of Logarithmic Functions**

The Chain Rule for Logarithmic Functions If u(x) is a differentiable function of x, then

$$\frac{d}{dx}[\ln u(x)] = \frac{u'(x)}{u(x)}$$

#### Example

Differentiate the function  $f(x) = \ln(x^2 + 1)$ .

# **Differentiation of Logarithmic Functions**

Example Differentiate the function  $f(x) = \ln(x^3 - 5x + 4)$ .

# **Differentiation of Logarithmic Functions**

#### Example

Find an equation for the tangent line to  $y = x + \ln x$  at the point where x = e.

## **Differentiation of Exponential Functions**

#### The Derivative of the Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$
 for every real number x

#### Example

Differentiate the function  $f(x) = \frac{e^x}{x}$ .

## **Differentiation of Exponential Functions**

The Chain Rule for Exponential Functions If u(x) is a differentiable function of x, then

$$\frac{d}{dx}e^{u(x)}=e^{u(x)}u'(x)$$

#### Example

Differentiate the function  $f(x) = xe^{2x}$ .

# **Differentiation of Exponential Functions**

#### Example

Find the largest and smallest values of the function  $F(x) = e^{x^2 - 2x}$  over the closed interval  $0 \le x \le 2$ .

# Logarithmic Differentiation

Differentiating a function that involves products, quotients, or powers can often be simplified by first taking the logarithm of the function.

- Step 1. Take logarithms of both sides of the expression for f(x) and simplify the resulting equation.
- Step 2. Use the chain rule to differentiate both sides.
- Step 3. Multiply both sides with f(x) to get f'(x).

# Logarithmic Differentiation

#### Example

Use logarithmic differentiation to find the derivative of

$$f(x)=\sqrt[4]{\frac{2x+1}{1-3x}}.$$

# Logarithmic Differentiation

#### Example

Use logarithmic differentiation to find the derivative of  $f(x) = \frac{e^{3x}(x^2 + 5)}{(1 - x)^5}.$