4.1 Exponential Functions

Definition of b^n for rational values of n (and $b > 0$)

- Integer Powers: If n is a positive integer,

$$b^n = \underbrace{b \cdot b \cdots b}_{n \text{ factors}}$$

- Fractional Powers: If n and m are positive integers,

$$b^{n/m} = (\sqrt[m]{b})^n = \sqrt[m]{b^n}$$

- Negative Powers: $b^{-n} = \dfrac{1}{b^n}$
- Zero Power: $b^0 = 1$

Exponential Functions

Definition

If b is a positive number other than 1 ($b > 0, b \neq 1$), there is a unique function called the exponential function with base b that is defined by

$$f(x) = b^x \quad \text{for all real number } x$$

Example

Sketch the graphs of $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$.

Exponential Functions

Basic Properties of Exponential Functions

For bases a, b and any real numbers x, y, we have

- The equality rule: $b^x = b^y$ if and only if $x = y$
- The product rule: $b^x b^y = b^{x+y}$
- The quotient rule: $\dfrac{b^x}{b^y} = b^{x-y}$
- The power rule: $(b^x)^y = b^{xy}$
- The multiplication rule: $(ab)^x = a^x b^x$
- The division rule: $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Exponential Functions

Example

Evaluate the given expression.

a. $8^{2/3}$

b. $(4^{2/3})(2^{2/3})$

c. $\dfrac{(3^{1.3})(3^{2.5})}{3^{3.2}}$

d. $(x^{3/2})^{-4/3}$

Exponential Functions

Example

Find all real numbers x that satisfy the given equation.

a. $3^x 2^{2x} = 144$

b. $2^{3-x} = 4^x$

The natural exponential base

The natural exponential base is the number e defined by

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$$\approx 2.71828\ldots$$