3.3. Curve Sketching

Vertical Asymptotes

The vertical line $x = c$ is a vertical asymptote of the graph of $f(x)$ if either

$$\lim_{x \to c^-} f(x) = +\infty \quad (\text{or } -\infty)$$

or

$$\lim_{x \to c^+} f(x) = +\infty \quad (\text{or } -\infty)$$

Vertical Asymptotes

Example

Determine all vertical asymptotes of the graph of

$$g(x) = \frac{2x^2 + 2x}{x^2 - 3x - 4}$$

Horizontal Asymptotes

The horizontal line $y = b$ is a horizontal asymptote of the graph of $f(x)$ if

$$\lim_{x \to -\infty} f(x) = b$$

or

$$\lim_{x \to +\infty} f(x) = b$$

Horizontal Asymptotes

Example

Determine all horizontal asymptotes of the graph of

$$g(x) = \frac{2x^2 + 2x}{x^2 - 3x - 4}$$

General Procedure for Sketching the Graph

Step 1. Find the domain of $f(x)$.

Step 2. Find and plot all intercepts.

Step 3. Determine all vertical and horizontal asymptotes and draw them.

Step 4. Find $f'(x)$ and determine the critical numbers and intervals of increase and decrease.

Step 5. Determine all relative extrema. Plot each relative maximum with a "cap" and each relative minimum with a "cup".

Step 6. Find $f''(x)$ and determine intervals of concavity and points of inflection. Plot inflection points with a "twist"

Step 7. Complete the sketch by joining the plotted points.

Curve Sketching

Example

Sketch the graph of $f(x) = \dfrac{4x}{(x+1)^2}$.

Curve Sketching

Example
Sketch the graph of $f(x) = \dfrac{x+3}{x-5}$.

Curve Sketching

Example

Sketch the graph of $f(x) = \dfrac{x+1}{x^2+x+1}$.