2.4. The Chain Rule

If y = f(u) is a differentiable function of u and u = g(x) is in turn a differentiable function of x, then the composite function f(g(x)) is a differentiable function of x whose derivative is given by the product

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

or, equivalently, by

$$\frac{dy}{dx} = f'(g(x))g'(x)$$

The Chain Rule

Example

Compute the derivative $\frac{dy}{dx}$ and simplify the answer if

$$y = u^2 - 3u + 4;$$
 $u = 1 - x^2$

The Chain Rule

Example

Compute the derivative

$$\left. e \left. \frac{dy}{dx} \right|_{x=\frac{1}{2}} \right|_{x=\frac{1}{2}}$$
 if

$$y = u^2 - 2u + 2; \quad u = \frac{1}{x}$$

The Chain Rule

Sometimes when dealing with a composite function y = f(g(x))it may help to think of *f* as the "outer" function and *g* as the "inner" function. Then the chain rule says that the derivative of y = f(g(x)) with respect to *x* is given by the derivative of the outer function evaluated at the inner function times the derivative of the inner function.

Example

$$h(x)=\sqrt{x^6-3x^2}$$

The General Power Rule

For any real number n and differentiable function h,

$$\frac{d}{dx}[h(x)]^n = n[h(x)]^{n-1}\frac{d}{dx}[h(x)]$$

Example

$$f(x) = (t^4 - 4t^2 + 4)^6$$

Combination with other rules

Example

$$f(x) = (2x+1)^4(3x-5)^2$$

Combination with other rules

Example

$$F(x) = rac{(1-2x)^3}{(3x+1)^2}$$

Higher derivatives

Example

Find the second derivative of the given function

$$y=(1-x^2)^3$$