2.2. Techniques of Differentiation

The Constant Rule

For any constant c, $\frac{d}{dx}[c] = 0$

The Power Rule

For any real number n, $\frac{d}{dx}[x^n] = nx^{n-1}$

Example

Differentiate the function $y = \sqrt{x^5}$.

The Constant Multiple Rule

If c is a constant and $f(x)$ is diffenentiable, then so is $cf(x)$ and

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Example

Differentiate the function $y = 2\sqrt[3]{x^4}$.

The Sum Rule

If $f(x)$ and $g(x)$ are diffenentiable, then so is their sum and

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

Example

Differentiate the function $y = \frac{2}{x} - \frac{2}{x^2} + \frac{1}{3x^3}$.

Differentiation of polynomials

Example

Differentiate the function $y = x^3(x^2 - 5x + 7)$.

Equation of tangent lines

Example

Find the equation of the line that is tangent to the graph of the function $y = \sqrt{x^3} - x^2 + \dfrac{16}{x^2}$ at the point $(4, -9)$.

Relative and Percentage Rate of Change

The relative rate of change of a quantity $Q(x)$ with respect to x is

$$\frac{Q'(x)}{Q(x)}$$

The corresponding percentage rate of change of $Q(x)$ with respect to x is

$$\frac{100Q'(x)}{Q(x)}$$

Relative and Percentage Rate of Change

Example

It is estimated that t years from now, the population of a certain town will be $P(t) = t^2 + 100t + 8,000$.

a. Express the percentage rate of change of the population as a function of t.

b. What will happen to the percentage rate of change of the population in the long run?

Rectilinear Motion

Motion of an object along a line is called rectilinear motion.
If the position at time t of an object moving along a straight line is give by $s(t)$, the the object has

$$\text{velocity} \quad v(t) = s'(t) = \frac{dx}{dt}$$

and

$$\text{acceleration} \quad a(t) = v'(t) = \frac{dv}{dt}.$$

The object is advancing when $v(t) > 0$,
retreating when $v(t) < 0$, and
stationary when $v(t) = 0$.

It is accelerating when $a(t) > 0$ and
decelerating when $a(t) < 0$.

Rectilinear Motion

Example

The position at time t of an object moving along a line is given by $s(t) = t^3 - 9t^2 + 15t + 25$.

a. Find the velocity of the object.

b. Find the total distance traveled by the object between $t = 0$ and $t = 6$.

c. Find the acceleration of the object and determine when the object is accelerating and decelerating between $t = 0$ and $t = 6$.