# 2.1 The Derivative

## The derivative of a function

The derivative of the function $f(x)$ with respect to $x$ is the function $f'(x)$ given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

The process of computing the derivative is called differentiation, and we say that $f(x)$ is differentiable at $x = c$ if $f'(c)$ exists.

## Example

Find the derivative of the function $f(x) = x^2 - 2x$.

# Slope as a Derivative

The slope of the tangent line to the curve $y = f(x)$ at the point $(c, f(c))$ is $m_{tan} = f'(c)$.

## Example

Find the equation of the tangent line to the curve $y = x^2 - 2x$ at the point where $x = -1$.

# Instantaneous Rate of Change as a Derivative

The rate of change of $f(x)$ with respect to $x$ when $x = c$ is given by $f'(c)$.

## Example

A toy rocket rises vertically in such a way that $t$ seconds after lift-off, it is

$$h(t) = -\frac{1}{2}t^2 + 20t$$

feet above ground.

a. What is the (instanteneous) velocity of the rocket at lift-off?

b. What is its velocity after 10 seconds?

# Significance of the sign of $f'(x)$

If the function $f$ is differentiable at $x = c$, then

$f$ is increasing at $x = c$ if $f'(c) > 0$

and

$f$ is decreasing at $x = c$ if $f'(c) < 0$

Example

c. At lift-off, is the rocket rising?

d. Is the rocket rising after 30 seconds?

# Derivative Notation

The derivative $f'(x)$ of $y = f(x)$ is sometimes written as

$$\frac{dy}{dx} \text{ or } \frac{df}{dx}$$

In this notation, $f'(c)$ is written as

$$\left.\frac{dy}{dx}\right|_{x=c} \text{ or } \left.\frac{df}{dx}\right|_{x=c}$$

## Example

Find the rate of change $\dfrac{dy}{dx}$ of $y = 5 - x^2$ at the point where $x = 2$.

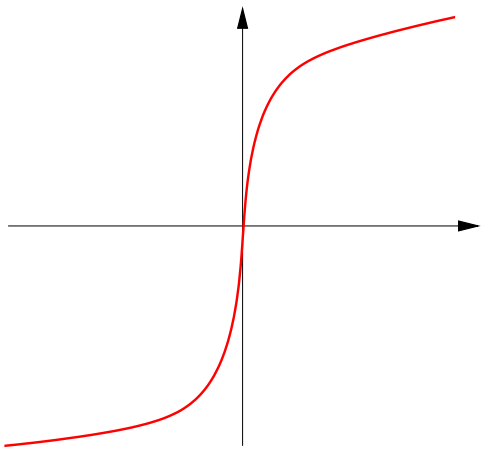
# Differentiability and Continuity

## Continuity of a differentiable function

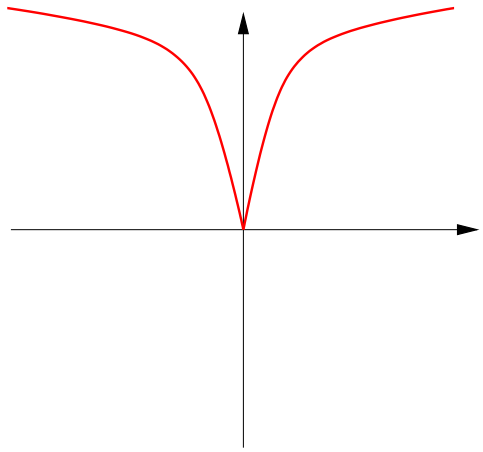
If the function $f(x)$ is differentiable at $x = c$, then it is also continuous at $x = c$.
The converse is not true.

## Example

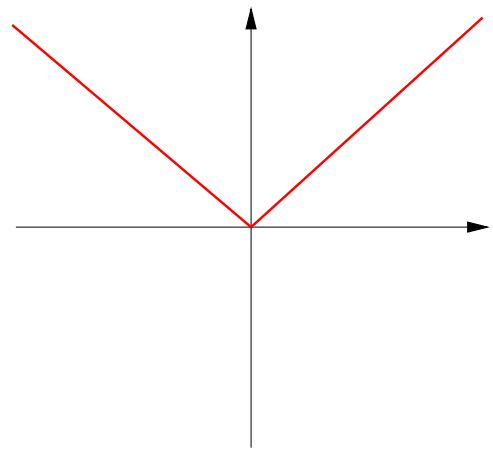
Continuous functions which are not differentiable



(a) Vertical tangent



(b) A cusp



(c) A “sharp point”