Definition

A linear function is a function that changes at a constant rate with respect to its independent variable.

- The graph of a linear function is a straight line.
- The equation of a linear function can be written as

$$y = mx + b$$

where *m* and *b* are constants.

Linear Functions

Definition The slope of the nonvertical line passing through the points (x_1, y_1) and (x_2, y_2) is the ratio

Slope =
$$\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Example

Find the slope of the line that passes through (5, -1) and (3, 3).

The Slope-Intercept Form

Definition (The Slope-Intercept Form) The equation

y = mx + b

is the equation of the line whose slope is m and whose y-intercept is (0, b).

Example

Find the equation of the line that passes through (5, -1) and (3, 3).

The Slope-Intercept Form

Example

Find the slope and y intercept of the line 5y - 3x = 4.

Horizontal and Vertical Lines

Example

Find the equation of the line that passes through (5, -1) and (3, -1).

Example

Find the equation of the line that passes through (5, -1) and (5, 1).

The Point-Slope Form

Definition (The Point-Slope Form)

The equation

 $y-y_0=m(x-x_0)$

is the equation of the line that passes through the point (x_0, y_0) and that has slope equal to *m*.

Example

Find the equation of the line that passes through (1,2) with slope $\frac{2}{3}$.

The Point-Slope Form

Example

Find the equation of the line that passes through (2,5) and (1,-2).

Practical Applications

Example

A certain car rental agency charges \$30 per day plus 55 cents per mile.

- a. Express the cost of renting a car from this agency for 1 day as a function of the number of miles driven and draw the graph.
- b. How much does it cost to rent a car for a 1-day trip of 250 miles?
- c. How many miles were driven if the daily rental cost was \$74?

Parallel and Perpendicular lines

Let m_1 and m_2 be the slopes of the nonvertical lines L_1 and L_2 . Then

- L_1 and L_2 are parallel if and only if $m_1 = m_2$.
- ► L_1 and L_2 are perpendicular if and only if $m_2 = \frac{-1}{m_1}$.

Example

Find the equation of the line that passes through (-3, 2) and parallel to the line x + 3y = 5.

Parallel and Perpendicular lines

Example

Find the equation of the line that passes through (1, 2) and perpendicular to the line x + 3y = 5.