1.1. Functions

Loosely speaking, a function consists of two sets and a rule that associates elements in one set with elements in the other.

Definition

- A function is a rule that assigns to each objects in a set A exactly one object in a set B.
- The set A is called the domain of the function.
- The set of assigned abjects in B is called the range.
- The value that the function f assigns to the number x in the domain is denoted by $f(x)$, which is often given by a formula, such as $f(x) = x^2 + 3$.
- If a function is given by an equation $y = f(x)$, then x is the independent variable and y is the dependent variable.

Example

Find $f(2)$ if $f(x) = x^2 + 3$.

Example

If $g(u) = (u + 1)^{3/2}$, find $g(0), g(-1)$, and $g(8)$.

Piecewise-defined function

Example

Find $h(2), h(1), h(-2)$ if

$$h(x) = \begin{cases} -2x + 4 & \text{if } x \leq 1 \\ x^2 + 1 & \text{if } x > 1 \end{cases}$$

Domain Convention

If a formula (or several formulas) is used to define a function f, then we assume that the domain of f to be the set of all numbers for which $f(x)$ is defined.

Example

Find the domain of the given functions.

a. $f(t) = \dfrac{t+3}{t^2 - t - 2}$

b. $h(x) = \sqrt{x^2 - 4}$

Functions used in Economics

- A demand function $p = D(x)$ is a function that relates the unit price p for a particular commodity to the number of units x demanded by consumers at that price.
- The total revenue is

$$\begin{aligned} R(x) &= \text{(number of items sold)(price per item)} \\ &= xp = xD(x) \end{aligned}$$

- If $C(x)$ is the total cost of producing the x units, then the profit derived from their sale is

$$P(x) = R(x) - C(x) = xD(x) - C(x).$$

Functions used in Economics

Example

Consumers will buy x thousand units of a particular kind of coffee maker when the unit price is

$$p = -0.27x + 51$$

dollars. The cost of producing the x thousand units is

$$C(x) = 2.23x^2 + 3.5x + 85$$

thousand dollars.

a. What are $D(x), R(x)$, and $P(x)$?

b. For what values of x is the production of the coffee maker profitable?

Functions used in Economics

Example

Suppose the total cost in dollars of manufacturing q units of a certain commodity is

$$C(q) = q^2 + 40q + 500.$$

a. Compute the cost of manufacturing 10 units.

b. Compute the cost of manufacturing the 10th unit.

Composition of functions

Definition

Given functions $f(u)$ and $g(x)$, the composition $f(g(x))$ is the function of x formed by substituting $u = g(x)$ for u in the formula for $f(u)$.

Example

Find the composite function $f(g(x))$, where $f(u) = u^2 + 3$ and $g(x) = x - 1$.

Composition of functions

Example

Find the composite functions $f(g(x))$ and $g(f(x))$, where $f(x) = x^2 + 3x + 1$ and $g(x) = 1 + x$, and find all (if any) values of x such that $f(g(x)) = g(f(x))$.

Composition of functions

Example

At a certain factory, the total cost of manufacturing q units during the daily production run is $C(q) = q^2 + q + 900$ dollars. On a typical workday, $q(t) = 25t$ units are manufactured during the first t hours of a production run.

a. Express the total manufacturing cost as a function of t.

b. How much will have been spent on production by the end of the third hour?

c. When will the total manufacturing cost reach $11,000?

Difference quotient

Definition

A difference quotient is an expression of the general form

$$\frac{f(x+h)-f(x)}{h}$$

where f is a function of x and h is a number.

Example

Find the difference quotient for $f(x) = 2x - x^2$.