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Solution of Quiz 3

1 Find the rate of change $\dfrac{dy}{dx}$ for the function $y = \dfrac{x^3+1}{x^2+1}$ when $x = 1$.

Solution. By the quotient rule, the derivative of $y = \dfrac{x^3+1}{x^2+1}$ with respect to x is given by

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left[\frac{x^3+1}{x^2+1}\right] \\ &= \frac{(x^2+1)\dfrac{d}{dx}[x^3+1] - (x^3+1)\dfrac{d}{dx}[x^2+1]}{(x^2+1)^2} \\ &= \frac{(x^2+1)(3x^2) - (x^3+1)(2x)}{(x^2+1)^2}\end{aligned}$$

Thus $\dfrac{dy}{dx}$ when $x = 1$ is

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{(1^2+1)(3(1)^2) - (1^3+1)(2(1))}{(1^2+1)^2} = \frac{2}{4} = \frac{1}{2}$$

2 Find the second derivative of the function $f(x) = x^6 - 2x^5 - 6x + 5$.

Solution. The first derivative is

$$\begin{aligned}f'(x) &= \frac{d}{dx}\left[x^6 - 2x^5 - 6x\right] \\ &= 6x^5 - 2(5x^4) - 6 \\ &= 6x^5 - 10x^4 - 6\end{aligned}$$

and then the second derivative is

$$\begin{aligned}f''(x) &= \frac{d}{dx}\left[f'(x)\right] \\ &= \frac{d}{dx}\left[6x^5 - 10x^4 - 6\right] \\ &= 6(5x^4) - 10(4x^3) \\ &= 30x^4 - 40x^3\end{aligned}$$