Lecturer: Sangwook Kim
Office : Science & Tech I, 226D
math.gmu.edu/~skim22

Solution of Quiz 3

1 Find the rate of change $\frac{dy}{dx}$ for the function $y = \frac{x^3+3}{x^2+1}$ when $x = 1$.

Solution. By the quotient rule, the derivative of $y = \frac{x^3+3}{x^2+1}$ with respect to x is given by

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left[\frac{x^3+3}{x^2+1}\right] \\ &= \frac{(x^2+1)\frac{d}{dx}[x^3+3] - (x^3+3)\frac{d}{dx}[x^2+1]}{(x^2+1)^2} \\ &= \frac{(x^2+1)(3x^2) - (x^3+3)(2x)}{(x^2+1)^2}\end{aligned}$$

Thus $\frac{dy}{dx}$ when $x = 1$ is

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{(1^2+1)(3(1)^2) - (1^3+3)(2(1))}{(1^2+1)^2} = \frac{-2}{4} = -\frac{1}{2}$$

2 Find the derivative of the function $f(x) = (x^3 - 4x + 4)^5$ when $x = 1$.

Solution. By the general power rule,

$$\begin{aligned}f'(x) &= \frac{d}{dx}\left[(x^3-4x+4)^5\right] \\ &= 5(x^3-4x+4)^4\frac{d}{dx}(x^3-4x+4) \\ &= 5(x^3-4x+4)^4(3x^2-4)\end{aligned}$$

Thus we have

$$f'(1) = 5[(1)^3 - 4(1) + 4]^4[3(1)^2 - 4] = 5(1)^4(-1) = -5.$$