Lecturer: Sangwook Kim
Office : Science & Tech I, 226D
math.gmu.edu/~skim22

Solution of Quiz 2

1 Decide if the following function is continuous at $x = 2$. Explain the reason.

$$f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x^2 - 3x + 2} & \text{if} \quad x < 2 \\[2ex] x^2 - x & \text{if} \quad x \geq 2 \end{cases}$$

Solution. We need to verify the three criteria for continuity are satisfied.

(a) $f(x)$ is defined; $f(x) = 2^2 - 2 = 2$.

(b) To decide whether the limit at $x = 2$ exists, we need to find two one-sided limits;

$$\lim_{x \to 2-} f(x) = \lim_{x \to 2-} \frac{x^2 - x - 2}{x^2 - 3x + 2} = \lim_{x \to 2-} \frac{(x-2)(x+1)}{(x-2)(x-1)} = \lim_{x \to 2-} \frac{x+1}{x-1} = \frac{3}{1} = 3$$
$$\lim_{x \to 2+} f(x) = \lim_{x \to 2+} (x^2 - x) = 2^2 - 2 = 2$$

Since two one-sided limits have different values, $\lim_{x \to 2} f(x)$ does not exist.

Therefore, $f(x)$ is not continuous at $x = 2$.

2 Find the slope of the tangent line to the curve $y = 2\sqrt{x^3} + \dfrac{1}{x^2}$ at the point where $x = 1$.

Solution. The derivative of the given function with respect to x is

$$\frac{dy}{dx} = \frac{d}{dx}\left[2x^{3/2} + x^{-2}\right] = 2\frac{d}{dx}\left[x^{3/2}\right] + \frac{d}{dx}\left[x^{-2}\right] = 2\left(\frac{3}{2}\right)x^{1/2} + (-2)x^{-3} = 3\sqrt{x} - \frac{2}{x^3}$$

Thus the slope of the tangent line to the given curve where $x = 1$ is given by

$$\left.\frac{dy}{dx}\right|_{x=1} = 3\sqrt{1} - \frac{2}{1^3} = 1$$