Solution of Quiz 1

1 Find all values of x such that f(g(x)) = g(f(x)), where $f(x) = x^2 + 2$ and g(x) = x - 1.

Solution. Replace x by g(x) = x - 1 in the formula for f(x) to get

$$f(g(x)) = g(x)^2 + 2 = (x - 1)^2 + 2 = x^2 - 2x + 3.$$

Similarly, we replace x by $f(x) = x^2 + 2$ in the formula for g(x) to get

$$g(f(x)) = f(x) - 1 = (x^2 + 2) - 1 = x^2 + 1.$$

By solving the equation f(g(x)) = g(f(x)), we get

$$x^{2} - 2x + 3 = x^{2} + 1$$
$$-2x = -2$$
$$x = 1$$

Thus f(g(x)) = g(f(x)) only when x = 1.

2 Find x and y intercepts of the function $f(x) = x^2 - 4x + 3$.

Solution. Since f(0) = 3, the y intercept is (0, 3). To find the x intercepts, solve the equation f(x) = 0. Factoring, we find that

$$x^{2} - 4x + 3 = 0$$
$$(x - 1)(x - 3) = 0$$

Thus, the x intercepts are (1,0) and (3,0).

3 Find the equation of the line that passes through the points (1,3) and (3,-1).

Solution. The slope of the given line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{3 - 1} = \frac{-4}{2} = -2.$$

Use the formula $y - y_1 = m(x - x_1)$ with m = -2 and $(x_1, y_1) = (1, 3)$ to get

$$y - 3 = -2(x - 1)$$

which can be rewritten as

$$y = -2x + 5.$$