## Solution of Quiz 0

**1** Find the interval or intervals consisting of all real numbers x that satisfy the given inequality.

(a)  $|x-1| \le 3$ 

SOLUTION. Rewrite the given inequality as

$$-3 \le x - 1 \le 3$$

and add 1 to each term to get

 $-2 \le x \le 4.$ 

(b) |2-x| > 4

SOLUTION. The given expression is true if and only if 2 - x > 4 or 2 - x < -4 which is equivalent to x < -2 or x > 6.

**2** Factor and simplify the given expressions as much as possible.

(a)  $3x^4 - 12x^2$ 

SOLUTION. Because the common factor of  $3x^4$  and  $12x^2$  is  $3x^2$ , we have

$$3x^4 - 12x^2 = 3x^2(x^2 - 4).$$

Using the formula  $A^2 - B^2 = (A + B)(A - B)$ , we get

$$3x^4 - 12x^2 = 3x^2(x^2 - 4) = 3x^2(x + 2)(x - 2).$$

(b) 
$$\frac{3(x-2)^2(x+1)^2 - 2(x-2)(x+1)^3}{(x-2)^2}$$

SOLUTION. First simplify the numerator to get

$$\frac{3(x-2)^2(x+1)^2 - 2(x-2)(x+1)^3}{(x-2)^2} = \frac{(x-2)(x+1)^2[3(x-2) - 2(x+1)]}{(x-2)^2}$$
$$= \frac{(x-2)(x+1)^2[3x-6-2x-2]}{(x-2)^2}$$
$$= \frac{(x-2)(x+1)^2(x-8)}{(x-2)^2}$$

and then cancel the common factor of x - 2 from the numerator and the denominator to get

$$\frac{3(x-2)^2(x+1)^2 - 2(x-2)(x+1)^3}{(x-2)^2} = \frac{(x+1)^2(x-8)}{x-2}.$$

**3** Solve the equation  $6x^2 - 7x - 3 = 0$  by factoring.

Solution. We want to find integers a, b, c, and d so that

$$6x^{2} - 7x - 3 = 0 = (ax + b)(cx + d)$$
  
=  $acx + (ad + bc)x + bd.$ 

so we must have

$$ac = 6$$
$$ad + bc = -7$$
$$bd = -3$$

Since a, b, c, and d are all integers, there are a limited number of possibilities for the choices; namely 8 choices for the pair a, c: 1, 6; 2, 3; 3, 2; 6, 1; -1, -6; -2, -3; -3, -2; and -6, -1 4 choices for the pair b, d: 1, -3; 3, -1; -1, 3; and -3, 1

There are 32 possible ways of forming the expression ad + bc. One can see that the condition ad + bc = -7 is satisfied when a = 2, b = -3, c = 3, and d = 1, so that

$$6x^2 - 7x - 3 = (2x - 3)(3x + 1).$$

Thus the solution for the given equation is  $x = \frac{3}{2}$  and  $x = -\frac{1}{3}$ .

4 Use the quadratic formula to solve the equation  $x^2 - 2x - 2 = 0$ .

SOLUTION. The given equation is a quadratic equation with a = 1, b = -2, and c = -2. Using the quadratic formula, we get

$$x = \frac{2 \pm \sqrt{2^2 - (-8)}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}.$$