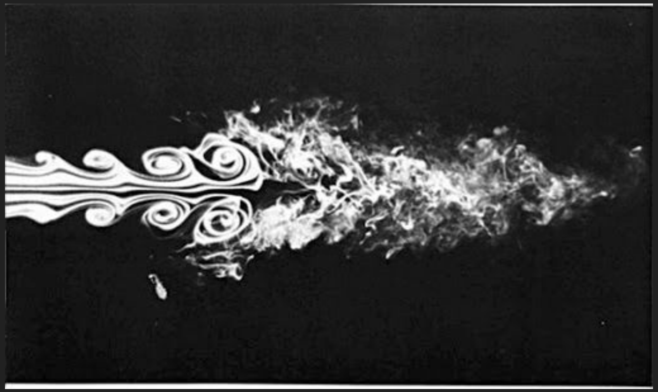
Linear algebra and computational mathematics

A brief presentation



My name is: Sean Carney

I am a Hedrick assistant adjunct professor (postdoc) at UCLA

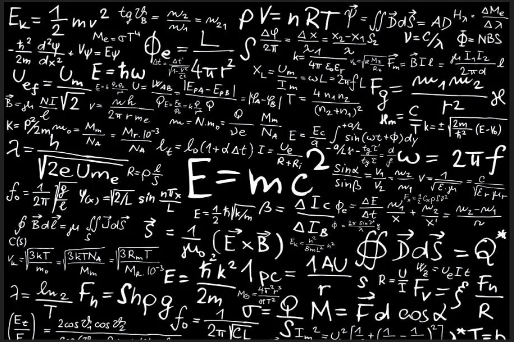
I research:

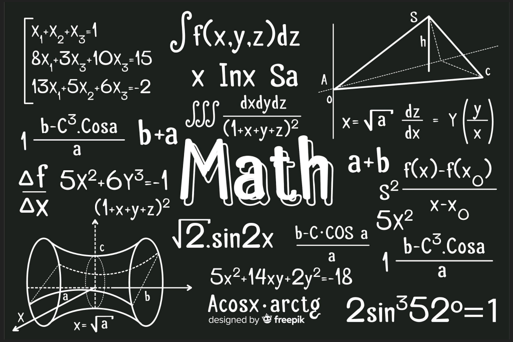
Multiscale modeling, analysis, and simulation

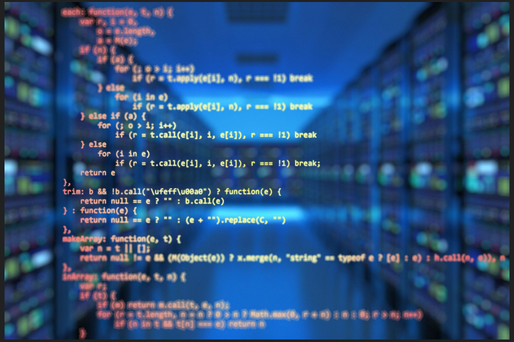
This research is a synthesis of three areas:

1) Modeling: physics
2) Analysis: mathematics
3) Simulation: computer science

(Basically, solving problems in physics on a computer using mathematics.)

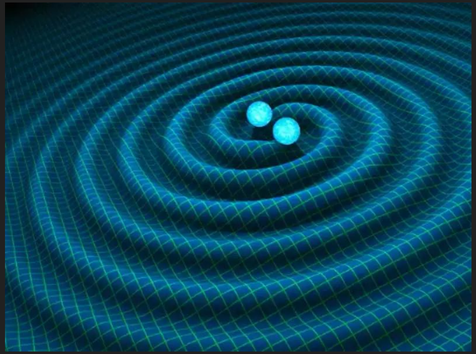


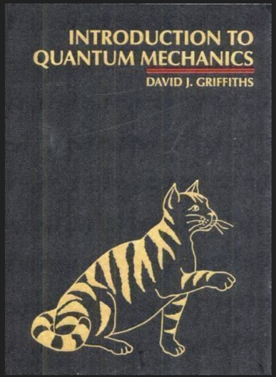




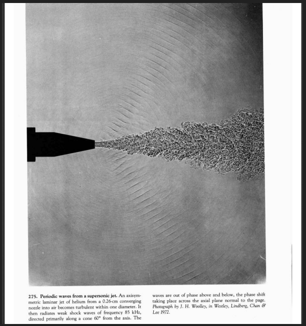
Multiscale?

We have physical models for “big” things (general relativity) and “small” things (quantum mechanics).





Sometimes the same physics occurs over a *wide range of scales in space and time*. Example → the weather



My path into mathematics:

A **direct** result of excellent mentors, teachers, and role models

1) High school: really curious about physics and philosophy
2) Undergrad at Univ. of Michigan: physics/math + computer science
3) PhD at UT Austin in mathematics

+2 other key experiences!

UNIVERSITY OF MICHIGAN

My path into mathematics:

A **direct** result of excellent mentors, teachers, and role models

Prof. Jean Krisch

Prof. Evelyn Lunasin

Prof. Divakar Viswanath

These three researchers (and many others) led me to cross paths with:

Prof. Smadar Karni

Dr. David Prigge

LSA MATHEMATICS
UNIVERSITY OF MICHIGAN

NSF Research Experiences For Undergraduates

My path into mathematics:

A **direct** result of excellent mentors, teachers, and role models

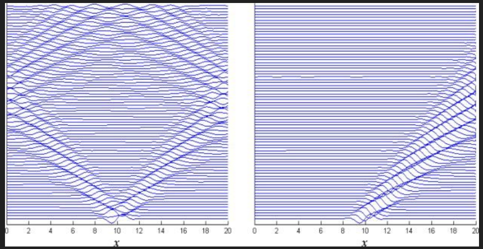
Key experience (1/2): Research Experience for Undergraduates (REU) program at U. Mich. summer before senior year.

Topic: numerical analysis for water waves.

M | LSA MATHEMATICS
UNIVERSITY OF MICHIGAN

NSF Research Experiences For Undergraduates

"You can never know too much linear algebra."
-Prof. Smadar Karni



I wouldn't have gone to graduate school in maths without this experience.

My path into mathematics:

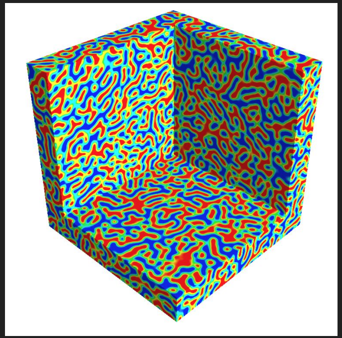
A **direct** result of excellent mentors, teachers, and role models

Key experience (2/2): graduate research assistant for two years at Lawrence Berkeley National Lab.

Topic: computational modeling of ionic liquids.

Dr. John Bell

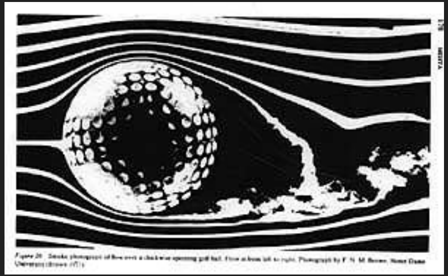
Dr. Ann Almgren



I wouldn’t be doing a postdoc in applied mathematics without this experience.

Linear algebra and my research:

Physical modeling always translates to mathematical equations (PDEs).



$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\rho g_z$$

Mathematical analysis used to *approximate* a (potentially complicated) solution with much simpler, *basis functions*, or building block functions.

$$u(x) \approx \sum_{i=1}^{N} c_i B_i(y)$$

This is a fundamental idea in linear algebra; it's very similar to working with unit vectors from physics:

$$\hat{\mathbf{i}} = \begin{bmatrix}1\\0\\0\end{bmatrix},\ \hat{\mathbf{j}} = \begin{bmatrix}0\\1\\0\end{bmatrix},\ \hat{\mathbf{k}} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

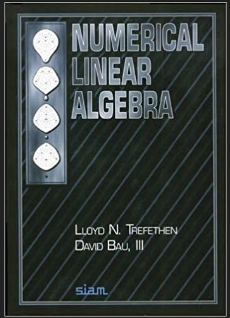
Linear algebra and my research:

The approximate, or *discrete*, physical model **inevitably** results in a linear system of algebraic equations that must be solved to get an answer.

$$u(x) \approx \sum_{i=1}^{N} c_i B_i(y) \longrightarrow Ac = u$$

N unknowns, N equations to solve. Not uncommon that N > 10,000 for example.

The mathematics of finding a solution vector ‘c’ *efficiently* is its own field--numerical linear algebra--using computers.



(many beautiful connections exist between numerical linear algebra and approximation theory, probability theory, etc.)

Takeaways:

Tirelessly pursue *what you find interesting*.

Get to know your instructors during class, office hours, etc. Find where their interests *intersect your own*.

It is never too early *and* it is never too late to involve yourself in a research project. REUs are great places to begin!

If you go to graduate school, consider a summer internship in industry or a government lab.

Above all, be kind to people.

“You can never know too much linear algebra.”