

## Math 315, Spring 2013, Discussion questions

**Come to class Monday ready to discuss these – either with questions or with ideas**

1. Try to write a careful statement of the negation of  $\lim_{n \rightarrow \infty} x_n = L$  in terms of the  $N, \epsilon$  definition of limit we developed in class.
2. (Textbook, theorem 1.2.1) This theorem says that every convergent sequence is a Cauchy sequence. Try to make good sense of this both informally and formally.
3. (Textbook, theorem 1.2.2) This theorem says that every Cauchy sequence is bounded. Try to make good sense of this both informally and formally. What underlying ideas come into play?
4. Try to formulate a careful statement of the definition of Cauchy sequence – meaning a statement of when a sequence is NOT a Cauchy sequence.
5. Suppose the sequence  $\{x_n\}$  has all positive terms and converges. Show that the limit cannot be negative using the definition of limit. Can it be 0? Then consider the situation where  $a < x_n < b$  for all  $n$  and the sequence converges. What can you say about the possible limit value(s)?
6. If a sequence  $\{x_n\}$  converges and, as collateral damage when Hansel and Gretel fight witches, the first 1000 terms are now set to the value  $-5$ , is the limit of the new sequence different from the original limit? Explain informally and formally.