Math 739: Differential Geometry
Problem Set #6
Due Thursday, April 15

1. Exercise 6.3 on page 126
2. Exercise 6.6 on page 132
3. Exercise 6-4 on page 151
4. Exercise 6-5 on page 151-152
5. Suppose that V, W are smooth vector fields on a smooth manifold M, and let $f : M \longrightarrow \mathbb{R}$ be a smooth function. Suppose that $p \in M$ is a critical point for f.

   (a) Prove that $V_p(Wf)$ is symmetric with respect to V and W.

   (b) Given local coordinates $\frac{\partial}{\partial x^1}, \cdots, \frac{\partial}{\partial x^n}$ around p, if the vector fields restricted to p are given by

   $$V_p = \sum_{i=1}^{n} v^i \frac{\partial}{\partial x^i}|_p \quad \text{and} \quad W_p = \sum_{i=1}^{n} w^i \frac{\partial}{\partial x^i}|_p,$$

   then

   $$V_p(Wf) = (v^1, \ldots, v^n) \left(\frac{\partial^2 f}{\partial x^i x^j}(p) \right) (w^1, \ldots, w^n)^T,$$

   where $\left(\frac{\partial^2 f}{\partial x^i x^j} \right)$ is the matrix of second derivatives (the Hessian).

6. EXTRA CREDIT (CHALLENGE) Exercise 6-6 on page 152.