

Math 739: Differential Geometry
Problem Set #3
Due Tuesday, February 23

1. Exercise 3.2 on page 66
2. Exercise 3-4 on page 79
3. Exercise 3-5 on page 79
4. Exercise 3-8 on page 79
5. Suppose $M = \mathbb{R}^2$, and let $B \subset M$ be the open ball of radius 1 centered at 0 and \overline{B} its closure. Prove that there is no smooth bump function for B whose support is contained in \overline{B} . Why is this relevant to Proposition 2.26 on page 55?
6. Let M be a topological manifold, and $\{p_1, p_2, p_3, \dots\}$ be an infinite sequence of points with $p_i \in M$. Suppose for every compact set N in M , N contains at most finitely many of these points. Clearly, M is not compact. Show that $\{p_i\}$ does not have a convergent subsequence. Show further that, if M is not compact, such a sequence always exists.