Math 739: Differential Geometry
Problem Set #1
Due Thursday, January 28

1. Consider $\pi : \mathbb{R}^{n+1}\backslash\{0\} \longrightarrow \mathbb{R}P^n$ given by taking an $n+1$-tuple to its linear span. Let $\tilde{U}_i = \{(y_1, \cdots, y_{n+1}) |\ y_i \neq 0\}$. Let $U_i = \pi(\tilde{U}_i)$. Show that U_i is open in the quotient topology given by π. Is the the topology generated by U_i enough to separate points in $\mathbb{R}P^n$? Prove that $\mathbb{R}P^n$ is Hausdorff.

2. What is $\mathbb{R}P^1$ and why?

3. Define M to be the quotient of S^n by the antipodal map $a : S^n \longrightarrow S^n$ given by

$$a(x) = -x$$

with the quotient topology obtained by using the topology on S^n we gave in class. In other words, $M := S^n/\sim$ where $x \sim y$ if and only if $a(x) = y$, i.e. $y = -x$. Prove that M is homeomorphic to $\mathbb{R}P^n$, where $\mathbb{R}P^n$ is defined as one-dimensional linear subspaces of $\mathbb{R}^{n+1}$.

4. p. 28, # 1-2

5. p. 28, #1-5

6. p. 28, #1-6

7. p. 28, #1-7