

Math 724: Problem Set 6

Due on 11.9.06

1. Show that $G = \{f_1 = x^3 - 2xy, f_2 = x^2y - 2y^2 + x\}$ is not a Gröbner basis for $I = \langle f_1, f_2 \rangle \subset k[x, y]$ in the grlex ordering.
2. Show that $G = \{y^3 - z^2, x^2 - y, xy - z, xz - y^2\}$ is a Gröbner basis for $I = \langle y - x^2, z - x^3 \rangle \subset k[x, y, z]$ in the grlex ordering..

3. Prove that

$$|Mon k[x_1, \dots, x_n]_{\leq m}| = \binom{m+n}{n}.$$

4. Calculate the Hilbert polynomial for $I = \langle x^3 - yz^2, y^4 - x^2yz \rangle \subset k[x, y, z]$.
5. The *Krull-dimension* of the ring R is the supremum of the lengths of chains of prime ideals in R . Let $I \subset k[x_1, x_2, x_3]$ be a monomial ideal. Describe the possible Krull dimensions of $k[x_1, x_2, x_3]/I$ for different I (prove your answer). Describe how this relates with the varieties that I represent.