Math 724: Problem Set 5

Due on 10.17.06

1. Let R be a Noetherian ring. Show that the following definitions are equivalent (this is an if-and-only-if).

   Definition A. Let I be an ideal. I is *primary* if, for all $ab \in I$, either $a \in I$ or $b^k \in I$ for some k.

   Definition B. Let I be an ideal. I is P-primary if for all $ab \in I$, if $a \notin P$, then $b \in I$. I is *primary* if it is P-primary for some prime ideal P.

2. Show that, in a Noetherian ring R, if P is nilpotent in R/I, then some power of P lies in I. Find an example that shows this is not true in a non-Noetherian ring, such as $\mathbb{R}[x_1, x_2, \dots]$ (infinitely many variables). We used this fact in the Noetherian case in our discussion of Theorem 3.1.

3. Exercise 3.6, p. 111. Note that answers are in the back of the book – make sure they are right and justify what you say!

4. Exercise 3.9(a), p. 111.

5. Exercise 3.10 (a), (b), p. 111.

6. Let R be a unique factorization domain. Show that R is integrally closed. That is, show that the normalization of R in its field of fractions S is just R itself. (Recall that the field of fractions S of R can be defined as the localization of R at $U = R - \{0\}$, since R is a domain, though you can use any definition that works for you).