Math 724: Problem Set 3

Due on 9.21.06

1. Let X be a subset of $\mathbb{A}^n$ over an algebraically closed field. Is $I(X)$ radical? Prove your answer.

2. Let $\mathbb{A}^n$ be affine space over an algebraically closed field. Give an example of a subset $X \subset \mathbb{A}^n$ such that $Z(I(X)) \neq X$. What condition would ensure equality? Prove your answer. You may use Hilbert Nullstellensatz, i.e. that
$$I(Z(I)) = rad\ I$$
but you may not use the fact that this gives a 1-1 bijection between algebraic sets and radical ideals unless you prove it.

3. Let k be an algebraically closed field. Show that all nonzero prime ideals in $k[x]$ are maximal. What does this mean about algebraic sets on $\mathbb{A}^1$? Would this be true if k were not algebraically closed? Assuming k is algebraically closed, find a nonzero prime ideal in $k[x_1, x_2]$ that is not maximal. What algebraic set does this correspond to?

4. 2.2 p. 79

5. Describe as explicitly as possible the $\mathbb{Z}$-module $Hom_{\mathbb{Z}}(\mathbb{Z}_p, \mathbb{Z}_m)$ where p is a prime number.

6. Describe as explicitly as possible $\mathbb{Z}_p \otimes_{\mathbb{Z}} \mathbb{Z}_m$ where p is a prime number.

7. (Extra Credit) 2.4 p. 79