Differential Topology Problem Set #3

Due Tuesday, March 8

1. Chapter 1, Section 4, #7
2. Chapter 1, Section 4, #8
3. Chapter 1, Section 4, #11(a)-(b)
4. Chapter 1, Section 5, #7
5. Chapter 1, Section 5, #10
6. Chapter 1, Section 6, #6. Read the definition of *contractible* from #4.
7. Chapter 1, Section 6, #7
8. Chapter 1, Section 7, #4
9. Chapter 1, Section 7, #6
10. Chapter 1, Section 7, #8 (just do a couple)
11. Chapter 1, Section 8, #1
12. Chapter 1, Section 8, #7

Additional problems for graduate students, or undergraduate extra credit

13.

14. Let $m^*(A)$ be the measure of A as defined in class. In other words, for all $\varepsilon > 0$, there is a countable set of rectangles $\{S_i\}$ such that $A \subset \cup_i S_i$ and

$$\sum_{i=1}^{\infty} \mathrm{vol}(S_i) - \varepsilon < m^*(A) \leq \sum_{i=1}^{\infty} \mathrm{vol}(S_i)$$

Find a (Cantor-like) subset A of the unit interval $[0, 1]$ with the properties:

(a) A is constructed by removing a countable number of intervals from $[0, 1]$ (open and/or closed)

(b) Between any two points $p, q \in A$, there is a point $b \in (p, q)$ not contained in A.

(c) $m^*(A) \neq 0$ and $m^*(A) < 1$.

Generalize this to obtain a set with these properties with arbitrary measure in $(0, 1)$. *Hint.* You will need to use convergence of series.

15. Find an example of a k-dimensional manifold such that $T(M)$ is not diffeomorphic to $M \times \mathbb{R}^k$, and prove your answer.