Differential Topology Problem Set #1

Due Thursday, Feb. 3

1. Prove that $S^n = \{x \in \mathbb{R}^{n+1} : |x| = 1\}$ is an $n-$dimensional manifold.

2. Section 1, #2

3. Section 1, #3

4. Section 1, #5

5. Suppose that U is an open set in $\mathbb{R}^n$. Prove that if $L : U \to \mathbb{R}^m$ is a linear map, then $dL_x = L$ for all $x \in U$. *Hint.* Suppose $0 \in U$ and try to prove the statement first at $x = 0$.

6. Section 2, #1

7. Section 2, #4

8. Section 2, #12

9. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be the function

$$f(x, y) = (x^2 - y, x + y, 5).$$

Write out df in matrix form.

10. Prove that if $\phi : U \to V$ is a diffeomorphism of open sets in $\mathbb{R}^n$, then

$$(d\phi)^{-1} = d(\phi^{-1}).$$

Additional problems for graduate students, or undergraduate extra credit

11. Section 1, #10

12. Section 1, #16

13. Section 2, #9(a-d)

14. Section 2, #10