Select Solutions Problem Set #10

1 Section 2.6

8 (a). As suggested in the text, we begin by finding the number of 3-digit positive even integers with no repeated digits that finish in 0. Since the first digit can be 1 through 9 (it can't be 0 since it is a 3-digit number), there are 9 possibilities for the first digit. The second digit has 10 possibilities (digits 0 through 9) but it can't be 0 because the last digit is 0, and it can't be whatever the first digit is, so there are only 8 possibilities. Thus there are $9 \cdot 8 = 72$ possibilities that end in 0.
8(b). Now suppose there is a non-zero units digit. Since we're counting even numbers, there are 4 possible units digits (2,4,6,8). After we choose the units digit, the hudreds digit can be any one of 8 possibilities – among the numbers 1 through 9, it has to avoid the units digit whatever that is. Finally for the middle term, it could be any one of 10 possibilities, 0 through 9, but it has to avoid both the hundreds and units digits that have already been chosen – so it really has only 8 possibilities. Thus the number of 3 digit numbers with no repeated digits and ending in a non-zero units digit is $4 \cdot 8 \cdot 8 = 256$.

We conclude that in total there are $72 + 256 = 328$ even 3-digit numbers with no repeated digits. We check this (as we are asked to do in the book) by counting this in another way. The number of even numbers with 3-digits and no repeats should be equal to the *total* number of positive 3-digit integers with no repeated digits, minus the number of *odd* positive 3-digit integers with no repeated digits. The total number is given by $9 \cdot 9 \cdot 8 = 648$ (since there are 9 choices for the hundreds digit, 1 through 9, and 9 choices for the tens digit, 0 through 9 but not equal to the hundreds digit, and then 8 choices for the last digit, since it is 0 through 9 but can't equal either of the first two). The number of odd numbers (3-digits, no repeats) is given by noting that the units digit can be one of 5 possibilities, 1,3,5,7,9, and then the the hundreds digit can be one of 8 possibilities, 1 through 9 and avoiding the units digit. Finally the tens digit can be one of 8 possibilities, 0 through 9 but avoiding whatever was the units digit and the hundreds digit. By the multiplication principle, there are $5 \cdot 8 \cdot 8 = 320$ such possibilities. At last we do our check. We expect that $\texttt{total} - \texttt{odd} = even$. Indeed, we see $648 - 320 = 328$, as desired. 15. There are 7

digits listed. Since each password has to use each number exactly once, the passwords must all be 7 digits long. Since the numbers cannot repeat but their order is important, we can start at the beginning, counting our possibilities, and use the multiplication principle. There are 7 choices for the first number, 6 remaining for the second number, 5 for the third, etc. In total there are $7! = 5040$ possibilities.

2 Section 3.1

4(b). The idea of the inverse set is that all the pairs (x, y) are switched – and now you have to have a new relation that describes how the second entry can be expressed as the first. To that end, if $(x, y) \in R_2$, then by definition, $y = -5x + 2$. If we switch these two entries, we have pairs (x, y) such that $x = -5y + 2$. We solve for y so the relation looks as if the second entry is a function of the first, obtaining $y = \frac{x-3}{-5} = \frac{3-x}{5}$. Therefore

$$R_2^{-1} = \left\{(x, y) \in \mathbb{R} \times \mathbb{R} : y = \frac{3-x}{5}\right\}.$$

5(g). We work from the inside out, and from the right to the left. First we consider $S \circ T$. Only 5 is in both the range of T (with the element $(3, 5)$) and the domain of S (with the element $(5, 5)$). Therefore, $S \circ T$ consists only of the pair $(3, 5)$. Now we compose the relation $S \circ T = \{(3, 5)\}$ with R and find that $(5, 2)$ is a pair in R, so the final result is that $R \circ (S \circ T) = \{(3, 2)\}$.

3 Section 3.2

1(h). This relation is not reflexive, since it is not always the case that xRx. For example, if $x = 6$, we see that $(6, 6)$ is not in this set. The relation is symmetric, since $x + y = 10$ implies $y + x = 10$, so if (x, y) is in the set, so is (y, x). (In other words, xRy holds if and only if yRx holds). Finally, this relation is not transitive, since $(4, 6)$ is in the set, $(6, 4)$ is in the set, but $(4, 4)$ is not in the set – said another way, we have $4R6$ and $6R4$ but not $4R4$, as would be proscribed by a relation R being transitive.

5(b) We prove this is an equivalence relation by proving that R is reflexive, symmetric, and transitive. Clearly R is reflexive, since for any natural number n, it has the same tens digit as itself, i.e. nRn. Next we check it is symmetric. If nRm for some natural numbers n and m, then they n has the same tens digit as m. But then m has the same tens digit as n, i.e. mRn. Finally, we check it is transitive. If nRm and mRk for natural numbers n, m, k, then n has the same tens digit as m, and m has the same as that of k, so the tens digit of n and k are equal, showing nRk, as desired. Therefore, this relation is an equivalence relation.

We are asked to find an element of $106/R$ that is less than 50: the number 6. An element of $106/R$ between 150 and 300: the number 208. An element of $106/R$ that is greater than 1000: the number 1002. Finally, three such elements in the class $635/R$ are 33, 231, and 1036, respectively. Note that there are many answers to this – you just need to have the same tens place digit as specified by the class.

7(b). This is reflexive (each vertex has an arrow to and from itself). This is symmetric because if ever there is an arrow from one element to another, there is also an arrow pointing in the opposite direction. Finally, the relation is transitive, because if you have an arrow from one vertex to another, and then a second arrow from the second vertex to a third, there is also an arrow from the first to the third. (Note that in this particular graph, two of the vertices will be the same for any such situation!).