MATH 113: MAPLE 1 Due: Thursday, October 23 in Recitation
Fall 2003 - Goldin Late assignments will not be accepted.

Instructions: You may work on this individually or in groups of not more than three people. If you work as a group, please hand in **only one copy with each person's name** on the paper. Choose the text mode under INSERT for your written answers.

1. (10 points) In this exercise, you will find the following limit, or determine that it does not exist:

$$\lim_{x \to 0} \frac{e^{\frac{2}{5}x} - 1 - \frac{2}{5}x}{x^2}.$$

   (a) Define the function

$$f(x) = \frac{e^{\frac{2}{5}x} - 1 - \frac{2}{5}x}{x^2}$$

   Note: e^x is exp(x).

   (b) Calculate a **table** of functional values as x approaches 0 from both sides. Create at least five values on each side of 0. Write a sentence about what you suspect the limit is based on these values.

   (c) **Graph** the function for values of x close enough to 0 (interpret 'close enough' as being able to determine the value of the limit to 3 decimal places). Remember here and in part (b) that the question is asking about the limit of this function as x approaches 0 so the function values not near x = 0 are irrelevant. Write a sentence about what you suspect the limit is based on this graph.

   (d) Use MAPLE's **limit** command to find the limit.

2. (10 points) Evaluate the following limit or determine that it does not exist

$$\lim_{x \to 0} \sin\left(\frac{1}{x^2}\right)$$

   using the same three methods outlined in problem 1 (b)-(d).

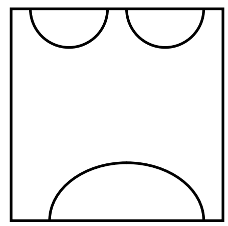
3. (10 points) Evaluate the following limit or determine that it does not exist

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x^2}\right)$$

   again using the same three methods outlined in problem 1 (b)-(d) Write a sentence or two describing how and why the result in this problem is different from that in problem 2. Note: Don't forget the * in your definition for multiplying x^2 and $\sin\left(\frac{1}{x^2}\right)$.

4. (10 points) Define two new functions g(x) = $e^{3\left(x-\frac{1}{2}\right)}$ and h(x) = cos (5x). Plot g(h(x)) and identify its domain and range. Plot h(g(x)) and identify its domain and range. Is the range of g(x) different from the range of g(h(x))? If so, explain why. Is the range of h(x) different from the range of h(g(x))? If so, explain why.

5. (10 points)
   (a) By plotting three parabolas on the same graph (use the window -4 ≤ x ≤ 4 and -4 ≤ y ≤ 4) create a "grumpy sleepy face." That is, stretch, reflect, and shift the parabola y = x^2 to get a mouth and two eyes that look something like the sketch:



   (b) Now put a "nose" inside by graphing a small circle around the origin. You may need to use the parametric equations of a circle with a small radius to complete this problem.

Hints for using MAPLE.

- To start MAPLE, from the osf1 prompt, type ‘xmaple’. Be sure to save your work often!!!

- Define a function f(x) = x^2 + 4x + 3:

  - f := x → x^2 + 4*x + 3;

- Find a functional value:

  - f(8);

- Evaluate as a floating point (decimal) number:

  - evalf(f(8));

- Display a graph:

  - plot (f(x), x=xmin..xmax, y=ymin..ymax);

    OR

  - plot (f, xmin..xmax, ymin..ymax);

  where you must supply numerical values of xmin, xmax, etc. The y-range is optional. The plot command is very fussy about the syntax used - - - be careful and very patient.
  Note that > plot (f, x=-1..1) and > plot (f(x), -1..1) will not work. Note that once the plot command is working, it is very easy to change the min and max values of x and y and to zoom in or out.

- Display multiple graphs in the same figure:

  - plot ({f(x),x+1,3*x^2}, xmin..xmax); OR

  - plot ({f,g,h}, xmin..xmax, ymin..ymax);

  where for the second case you must have already defined *g* and *h* as well.

- The limit command $\lim_{x \to a}$ f(x):

  - limit (f(x), x=a);

where you must supply the value of a and have already defined f.

- Other notes: π is Pi, e^x is exp(x), ln *x* is ln(x), sin *x* is sin(x), cos *x* is cos(x). A composite function f(g(x) is defined by (f@g)(x). **Do not forget the ‘;’ at the end of each command.** Help with a specific command can be found by typing a ‘?’ in front of the command. For example, > ?plot will give help with the plot command.