Negating $p \rightarrow q$

$\sim (p \rightarrow q)$ what does this mean?

"If $3$ is odd, then $5^2 = 26$.

If $\frac{3}{p}$, then $\frac{5^2}{q}$

$p \rightarrow q$

$\sim (p \rightarrow q)$ How do you say this in words?

Recall $p \rightarrow q \iff \sim q \rightarrow \sim p$

$\iff \sim q \lor \sim p$?

NO

\[
p \rightarrow q \iff \sim q \lor \sim p
\]

\[
\sim (p \rightarrow q) \iff \sim (\sim q \lor \sim p)
\]

\[
\sim (p \rightarrow q) \iff \sim p \land q
\]

"3 is odd and $5^2 \neq 26$."
"The person $x$ is nice"

Open sentence. It becomes a statement when we specify $x$.

"The person John is nice"
"The person Maria is nice"
etc.

Need to specify a universe of values that $x$ can take on.
For example, $U = \{\text{people in this classroom}\}$.

$p(x) = \text{"The integer } x \text{ is a square number.\"}$

If $U = \{1, 2, 3, 4, 5, \ldots\}$

Then $p(2)$ is false
$p(1)$ is true
$p(5)$ is false
$p(25)$ is true...

"$p(x)$ is true for every element $x \in U$\"
FALSE!

"$p(x)$ is true for some element $x \in U$\"
TRUE!
For every element \( x \) in \( U \)
\[ \forall x \in U \]

There exists an element \( x \in U \)
\[ \exists x \in U \]
such that...

\[ \exists x \in U \] \[ x \in \{1, 2, 3, 4, \ldots \} \]

\[ \exists x \in U, \; x > 5. \]
"There exists an \( x \) in \( U \) such that \( x > 5 \)."

This doesn't specify \( x \), but it is a statement (in this case, it's true).

\[ \forall x \in U, \; x = 5. \]
"For all elements \( x \) in \( U \), \( x = 5 \)."
FALSE.

**Example**
\[ \forall x \in U, \; (x-3)(x+4) > 0 \]

\( (x-3)(x+4) \neq 0 \)

\[ \text{FALSE.} \]

(When \( x = 3 \), \( (x-3)(x+4) \neq 0 \))

\[ \rightarrow \exists x \in U, \; (x-3)(x+4) \neq 0. \]

This is true.
Example

\[ \exists x \in U \quad x^2 = 1 \quad \text{TRUE} \]

\[ \exists x \in U \quad x^2 = -1 \quad \text{FALSE} \]

Universe matters!

\[ \exists x \in U \quad x < 0 \quad \text{FALSE} \]

proved it's false by showing

\[ \forall x \in U, \quad x \neq 0 \quad \text{TRUE} \]

Negate the statement

\[ \forall x \in U, \quad p(x) \quad \text{must be specified} \]

Whole thing is statement \((p(x) \text{ is not a statement})\).

\[ \neg (\forall x \in U, \ p(x)) \iff \exists x \in U, \ \neg p(x) \]

\[ \neg (\exists x \in U, \ p(x)) \iff \forall x \in U, \ \neg p(x) \]

De Morgan's Laws

\( \forall, \exists \) are "predicates."
For all $G$

"GMU students are broke and hungry."

$$U = \exists \text{GMU students} \exists \text{x}$$

$$p(x) = "x \text{ is broke."}$$

$$q(x) = "x \text{ is hungry."}$$

$$\forall x \in U, p(x) \land q(x).$$

Negate the statement:

"Some GMU students are not broke or not hungry."

$$\exists x \in U \sim (p(x) \land q(x))$$

$$\exists x \in U \sim p(x) \lor \sim q(x)$$

**MORE VARIABLES!**

Let $U = \exists \text{GMU students} \exists x$

$$p(x, y) = "x \text{ has higher cholesterol than y."}$$

$$q(x, y) = "x \text{ has a nicer car than y."}$$
What, in English, does this mean? 

\[ \forall x \in U \exists y \in U \left[ p(x, y) \rightarrow q(x, y) \right] \]

\[ p(x, y) \rightarrow q(x, y) \quad \text{"If } x \text{ has higher cholesterol than } y, \text{ then } x \text{ has a nicer car than } y.” \]

For every GMU student \( x \), there is a GMU student \( y \) such that if \( x \) has higher cholesterol, then \( x \) has a nicer car.”
Pick student A

There is a student B s.t.
cholesterol (A) > chol (B)

\[ \rightarrow \text{car}(A) \text{ nicer than car}(B) \]

Suppose we pick A, and I find a student B whose chol.
is higher than A.

Then we've satisfied the condition.