Weighted Composition Operators on the Bloch Space on a Bounded Homogeneous Domain

Robert F. Allen*
Flavia Colonna
{rallen2, fcolonna}@gmu.edu

Department of Mathematical Sciences
George Mason University

IWOTA XIX
July 25, 2008

Motivation

In their 2004 paper, Ohno & Zhao characterized the bounded and compact weighted composition operators

$$W_{\psi,\varphi}(f) = \psi(f \circ \varphi)$$

on the Bloch space and little Bloch space of the unit disk $\mathbb{D}$.

Theorem (Ohno & Zhao, 2004)

Let $\psi \in H(\mathbb{D}, \mathbb{C})$ and $\varphi \in H(\mathbb{D}, \mathbb{D})$. Then $W_{\psi,\varphi}$ is bounded on the Bloch space $\mathcal{B}(\mathbb{D})$ if and only if the following are satisfied:

(i) $\displaystyle\sup_{z\in\mathbb{D}} (1-|z|^2)\left|\psi'(z)\right| \log\left(\frac{2}{1-|\varphi(z)|^2}\right) < \infty;$

(ii) $\displaystyle\sup_{z\in\mathbb{D}} \frac{1-|z|^2}{1-|\varphi(z)|^2}\left|\psi(z)\varphi'(z)\right| < \infty.$

Research Goals

We are investigating multiplication, composition and weighted composition operators on the Bloch space on a class of domains in $\mathbb{C}^n$ which include $\mathbb{B}_n$ and $\mathbb{D}^n$.

In particular, we are trying to answer the fundamental questions:

1. What symbols induce bounded operators?
2. What are estimates on the norm of the bounded operators?
3. What symbols induce compact operators?
4. What symbols induce isometric operators?
5. What are the spectra of the bounded operators?

Definitions & Notation

- The multiplication and composition operators are defined as

$$M_\psi(f) = \psi f,$$
$$C_\varphi(f) = f \circ \varphi.$$

Definitions & Notation

- The multiplication and composition operators are defined as

$$M_\psi(f) = \psi f,$$
$$C_\varphi(f) = f \circ \varphi.$$

- A domain $D \subseteq \mathbb{C}^n$ is called homogeneous if $\operatorname{Aut}(D)$ acts transitively on D.

Definitions & Notation

- The multiplication and composition operators are defined as

$$M_\psi(f) = \psi f,$$
$$C_\varphi(f) = f \circ \varphi.$$

- A domain $D \subseteq \mathbb{C}^n$ is called homogeneous if $\text{Aut}\,(D)$ acts transitively on D.
- A domain $D \subseteq \mathbb{C}^n$ is called symmetric if for every $z_0 \in D$, there exists an involution $\phi \in \text{Aut}\,(D)$ for which z_0 is an isolated fixed point.

Definitions & Notation

- The multiplication and composition operators are defined as

$$M_\psi(f) = \psi f,$$
$$C_\varphi(f) = f \circ \varphi.$$

- A domain $D \subseteq \mathbb{C}^n$ is called homogeneous if $\mathrm{Aut}\,(D)$ acts transitively on D.
- A domain $D \subseteq \mathbb{C}^n$ is called symmetric if for every $z_0 \in D$, there exists an involution $\phi \in \mathrm{Aut}\,(D)$ for which z_0 is an isolated fixed point.
- Each bounded homogeneous domain has a canonical Möbius invariant metric, the Bergman metric, denoted by $H_z(\cdot,\cdot)$.

More Definitions & Notation (almost done)

- The Bloch space on a bounded homogeneous domain D is defined as

$$\mathcal{B}(D) = \left\{ f \in H(D, \mathbb{C}) : \sup_{z \in D} Q_f(z) < \infty \right\},$$

where

$$Q_f(z) = \sup_{u \in \mathbb{C}^n \setminus \{0\}} \frac{|(\nabla f)(z) \cdot u|}{H_z(u, \overline{u})^{1/2}}.$$

More Definitions & Notation (almost done)

- The Bloch space on a bounded homogeneous domain D is defined as

$$\mathcal{B}(D) = \left\{ f \in H(D, \mathbb{C}) : \sup_{z \in D} Q_f(z) < \infty \right\},$$

where

$$Q_f(z) = \sup_{u \in \mathbb{C}^n \setminus \{0\}} \frac{|(\nabla f)(z) \cdot u|}{H_z(u, \overline{u})^{1/2}}.$$

- For a bounded symmetric domain D, the closure of the polynomials in $\mathcal{B}(D)$ is a subspace called the little Bloch space, denoted by $\mathcal{B}_0(D)$.

More Definitions & Notation (almost done)

- The Bloch space on a bounded homogeneous domain D is defined as

$$\mathcal{B}(D) = \left\{ f \in H(D, \mathbb{C}) : \sup_{z \in D} Q_f(z) < \infty \right\},$$

where

$$Q_f(z) = \sup_{u \in \mathbb{C}^n \setminus \{0\}} \frac{|(\nabla f)(z) \cdot u|}{H_z(u, \overline{u})^{1/2}}.$$

- For a bounded symmetric domain D, the closure of the polynomials in $\mathcal{B}(D)$ is a subspace called the little Bloch space, denoted by $\mathcal{B}_0(D)$.

- The normalized unit ball of $\mathcal{B}(D)$ (or $\mathcal{B}_0(D)$) is defined as

$$\mathcal{B}_1(D) = \{ f \in \mathcal{B} (\text{or } \mathcal{B}_0) : f(0) = 0 \text{ and } ||f||_{\mathcal{B}} \leq 1 \}.$$

Multiplication Operators

What is known about the multiplication operator on the Bloch space?

1. Characterization of bounded multiplication operators on the Bloch space of $\mathbb{D}$.
2. Characterization of bounded multiplication operators on the Bloch space of $\mathbb{B}_n$ under an equivalent norm.
3. Characterization of compact multiplication operators on the Bloch space of $\mathbb{D}$.

Multiplication Operators

What is known about the multiplication operator on the Bloch space?

1. Characterization of bounded multiplication operators on the Bloch space of $\mathbb{D}$.
2. Characterization of bounded multiplication operators on the Bloch space of $\mathbb{B}_n$ under an equivalent norm.
3. Characterization of compact multiplication operators on the Bloch space of $\mathbb{D}$.

Theorem (Brown & Shields, 1990)

If $\psi \in H(\mathbb{D}, \mathbb{C})$, then the following are equivalent:

1. *M_ψ is bounded on $\mathcal{B}(\mathbb{D})$;*
2. *M_ψ is bounded on $\mathcal{B}_0(\mathbb{D})$;*
3. *$\psi \in H^\infty(\mathbb{D})$ and*

$$\left|\psi'(z)\right| \leq O\left(\frac{1}{(1-|z|)\log\frac{1}{1-|z|}}\right).$$

Multiplication Operators

What is known about the multiplication operator on the Bloch space?

1. Characterization of bounded multiplication operators on the Bloch space of $\mathbb{D}$.
2. Characterization of bounded multiplication operators on the Bloch space of $\mathbb{B}_n$ under an equivalent norm.
3. Characterization of compact multiplication operators on the Bloch space of $\mathbb{D}$.

Theorem (Zhu, 2004)

If $\psi \in H(\mathbb{B}_n, \mathbb{C})$, then the following are equivalent:

1. *M_ψ is bounded on $\mathcal{B}(\mathbb{B}_n)$.*
2. *M_ψ is bounded on $\mathcal{B}_0(\mathbb{B}_n)$.*
3. *$\psi \in H^\infty(\mathbb{B}_n)$ and*

$$(1 - |z|)^2 \, |\nabla \psi(z)| \log \frac{1}{1 - |z|^2} < \infty.$$

Multiplication Operators

What is known about the multiplication operator on the Bloch space?

1. Characterization of bounded multiplication operators on the Bloch space of $\mathbb{D}$.
2. Characterization of bounded multiplication operators on the Bloch space of $\mathbb{B}_n$ under an equivalent norm.
3. Characterization of compact multiplication operators on the Bloch space of $\mathbb{D}$.

Theorem (Ohno & Zhao, 2004)

If $\psi \in H(\mathbb{D}, \mathbb{C})$, then the following are equivalent:

1. *M_ψ is a compact operator on $\mathcal{B}(\mathbb{D})$.*
2. *M_ψ is a compact operator on $\mathcal{B}_0(\mathbb{D})$.*
3. *$\psi = 0$.*

Multiplication Operators on Bounded Homogeneous Domains

Definition

Let D be a bounded homogeneous domain in $\mathbb{C}^n$. For $z \in D$, define

$$\omega(z) = \sup_{f \in \mathcal{B}_1} |f(z)|,$$

$$\sigma_\psi = \sup_{z \in D} \omega(z) Q_\psi(z).$$

Multiplication Operators on Bounded Homogeneous Domains

Definition

Let D be a bounded homogeneous domain in $\mathbb{C}^n$. For $z \in D$, define

$$\omega(z) = \sup_{f \in \mathcal{B}_1} |f(z)|,$$

$$\sigma_\psi = \sup_{z \in D} \omega(z) Q_\psi(z).$$

Facts.

1. $\omega(z) \leq \rho(0, z)$ for all $z \in D$ where ρ is the Bergman distance function.

Multiplication Operators on Bounded Homogeneous Domains

Definition

Let D be a bounded homogeneous domain in $\mathbb{C}^n$. For $z \in D$, define

$$\omega(z) = \sup_{f \in \mathcal{B}_1} |f(z)|,$$

$$\sigma_\psi = \sup_{z \in D} \omega(z) Q_\psi(z).$$

Facts.

1. $\omega(z) \leq \rho(0, z)$ for all $z \in D$ where ρ is the Bergman distance function.
2. $\omega(z) = \sup_{f \in \mathcal{E}} |f(z)|$ where $\mathcal{E}$ is the set of extreme points of $\mathcal{B}_1(D)$.

Multiplication Operators on Bounded Homogeneous Domains

Definition

Let D be a bounded homogeneous domain in $\mathbb{C}^n$. For $z \in D$, define

$$\omega(z) = \sup_{f \in \mathcal{B}_1} |f(z)|,$$

$$\sigma_\psi = \sup_{z \in D} \omega(z) Q_\psi(z).$$

Facts.

1. $\omega(z) \leq \rho(0, z)$ for all $z \in D$ where ρ is the Bergman distance function.
2. $\omega(z) = \sup_{f \in \mathcal{E}} |f(z)|$ where $\mathcal{E}$ is the set of extreme points of $\mathcal{B}_1(D)$.
3. If $D = \mathbb{B}_n$, then $\omega(z) = \rho(0, z) = \frac{1}{2} \log \frac{1+||z||}{1-||z||}$ for all $z \in \mathbb{B}_n$.

Results for Multiplication Operators on Bounded Homogeneous Domains

If D is a bounded homogeneous domain in $\mathbb{C}^n$ and $\psi \in H(D, \mathbb{C})$, then

1. M_ψ is bounded on $\mathcal{B}(D)$ if and only if $\psi \in H^\infty(D)$ and $\sigma_\psi < \infty$.
2. $\max\{||\psi||_{\mathcal{B}}, ||\psi||_\infty\} \leq ||M_\psi|| \leq \max\{||\psi||_{\mathcal{B}}, ||\psi||_\infty + \sigma_\psi\}$.
3. $\sigma(M_\psi) = \overline{\psi(D)}$.

Results for Multiplication Operators on Bounded Homogeneous Domains

If D is a bounded homogeneous domain in $\mathbb{C}^n$ and $\psi \in H(D, \mathbb{C})$, then

1. M_ψ is bounded on $\mathcal{B}(D)$ if and only if $\psi \in H^\infty(D)$ and $\sigma_\psi < \infty$.
2. $\max\{||\psi||_{\mathcal{B}}, ||\psi||_\infty\} \leq ||M_\psi|| \leq \max\{||\psi||_{\mathcal{B}}, ||\psi||_\infty + \sigma_\psi\}$.
3. $\sigma(M_\psi) = \overline{\psi(D)}$.

If D is a bounded symmetric domain in $\mathbb{C}^n$ and $\psi \in H(D, \mathbb{C})$, then

1. M_ψ is bounded on $\mathcal{B}_0(D)$ if and only if $\psi \in H^\infty(D)$ and $\sigma_\psi < \infty$.
2. M_ψ is an isometry on $\mathcal{B}(D)$ if and only if ψ is a constant function of modulus 1.

Composition Operators on Bounded Homogeneous Domains

Definition

For $z \in D$, $\varphi \in H(D, D)$, and $J\varphi$ the Jacobian matrix of φ,

$$B_\varphi(z) = \sup_{u \in \mathbb{C}^n \setminus \{0\}} \frac{H_{\varphi(z)}\left(J\varphi(z)u, \overline{J\varphi(z)u}\right)^{1/2}}{H_z(u, \overline{u})^{1/2}},$$

$$B_\varphi = \sup_{z \in D} B_\varphi(z).$$

Composition Operators on Bounded Homogeneous Domains

Definition

For $z \in D$, $\varphi \in H(D,D)$, and $J\varphi$ the Jacobian matrix of φ,

$$B_\varphi(z) = \sup_{u \in \mathbb{C}^n \setminus \{0\}} \frac{H_{\varphi(z)}\left(J\varphi(z)u, \overline{J\varphi(z)u}\right)^{1/2}}{H_z(u,\overline{u})^{1/2}},$$

$$B_\varphi = \sup_{z \in D} B_\varphi(z).$$

Theorem

If D is a bounded homogeneous domain in $\mathbb{C}^n$ and $\varphi \in H(D,D)$, then C_φ is bounded on $\mathcal{B}(D)$ and

$$\max\{1, \omega(\varphi(0))\} \leq ||C_\varphi|| \leq \max\{1, \omega(\varphi(0)) + B_\varphi\}.$$

Weighted Composition Operators on Bounded Homogeneous Domains

As was the case on the unit disk, we wish to characterize the bounded weighted composition operators on the Bloch space of a bounded homogeneous domain in terms of the following two quantities:

$$\sigma_{\psi,\varphi} = \sup_{z \in D} \omega(\varphi(z)) Q_\psi(z)$$

$$\tau_{\psi,\varphi} = \sup_{z \in D} |\psi(z)|\, B_\varphi(z).$$

Weighted Composition Operators on Bounded Homogeneous Domains

As was the case on the unit disk, we wish to characterize the bounded weighted composition operators on the Bloch space of a bounded homogeneous domain in terms of the following two quantities:

$$\sigma_{\psi,\varphi} = \sup_{z\in D} \omega(\varphi(z))Q_\psi(z)$$
$$\tau_{\psi,\varphi} = \sup_{z\in D} |\psi(z)|\, B_\varphi(z).$$

Conjecture

Let D be a bounded homogeneous domain. If $\psi \in H(D,\mathbb{C})$ and $\varphi \in H(D,D)$, then $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(D)$ if and only if $\psi \in \mathcal{B}(D)$ and both $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite.

What We Can Prove

Theorem

Let D be a bounded homogeneous domain, $\psi \in H(D, \mathbb{C})$ and $\varphi \in H(D, D)$. If $\psi \in \mathcal{B}(D)$ and both $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite, then $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(D)$.

What We Can Prove

Theorem

Let D be a bounded homogeneous domain, $\psi \in H(D, \mathbb{C})$ and $\varphi \in H(D, D)$. If $\psi \in \mathcal{B}(D)$ and both $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite, then $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(D)$.

Theorem

Let D be a bounded homogeneous domain, $\psi \in H(D, \mathbb{C})$ and $\varphi \in H(D, D)$. If $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(D)$, then $\psi \in \mathcal{B}(D)$ and $\sigma_{\psi,\varphi}$ is finite if and only if $\tau_{\psi,\varphi}$ is finite.

What We Can Prove

Theorem

Let D be a bounded homogeneous domain, $\psi \in H(D, \mathbb{C})$ and $\varphi \in H(D, D)$. If $\psi \in \mathcal{B}(D)$ and both $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite, then $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(D)$.

Theorem

Let D be a bounded homogeneous domain, $\psi \in H(D, \mathbb{C})$ and $\varphi \in H(D, D)$. If $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(D)$, then $\psi \in \mathcal{B}(D)$ and $\sigma_{\psi,\varphi}$ is finite if and only if $\tau_{\psi,\varphi}$ is finite.

We feel that more investigation into the quantities $\omega(z)$ and/or $B_\varphi(z)$ will allow for the conjecture to be proved.

Weighted Composition Operators on $\mathbb{B}_n$

Theorem

If $\psi \in H(\mathbb{B}_n, \mathbb{C})$ and $\varphi \in H(\mathbb{B}_n, \mathbb{B}_n)$, then $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(\mathbb{B}_n)$ if and only if $\psi \in \mathcal{B}(\mathbb{B}_n)$ and both $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite.

Weighted Composition Operators on $\mathbb{B}_n$

Theorem

If $\psi \in H(\mathbb{B}_n, \mathbb{C})$ and $\varphi \in H(\mathbb{B}_n, \mathbb{B}_n)$, then $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(\mathbb{B}_n)$ if and only if $\psi \in \mathcal{B}(\mathbb{B}_n)$ and both $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite.

Theorem

If $\psi \in H(\mathbb{B}_n, \mathbb{C})$ and $\varphi \in H(\mathbb{B}_n, \mathbb{B}_n)$, then $W_{\psi,\varphi}$ is bounded on $\mathcal{B}_0(\mathbb{B}_n)$ if and only if the following conditions are satisfied:

1. $\psi \in \mathcal{B}_0(\mathbb{B}_n)$;
2. $\sigma_{\psi,\varphi}$ *and* $\tau_{\psi,\varphi}$ *are finite;*
3. $\displaystyle\lim_{||z||\to 1} |\psi(z)| \sup_{u\in\mathbb{C}^n\setminus\{0\}} \frac{||J\varphi(z)u||}{H_z(u,\overline{u})^{1/2}} = 0.$

Weighted Composition Operators on $\mathbb{B}_n$

Theorem

If $\psi \in H(\mathbb{B}_n, \mathbb{C})$ and $\varphi \in H(\mathbb{B}_n, \mathbb{B}_n)$, then $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(\mathbb{B}_n)$ if and only if $\psi \in \mathcal{B}(\mathbb{B}_n)$ and both $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite.

Theorem

If $\psi \in H(\mathbb{B}_n, \mathbb{C})$ and $\varphi \in H(\mathbb{B}_n, \mathbb{B}_n)$, then $W_{\psi,\varphi}$ is bounded on $\mathcal{B}_0(\mathbb{B}_n)$ if and only if the following conditions are satisfied:

1. $\psi \in \mathcal{B}_0(\mathbb{B}_n)$;
2. $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite;
3. $\displaystyle \lim_{||z|| \to 1} |\psi(z)| \sup_{u \in \mathbb{C}^n \setminus \{0\}} \frac{||J\varphi(z)u||}{H_z(u, \overline{u})^{1/2}} = 0.$

For $n = 1$, this was obtained by Ohno & Zhao.

Operator Norm Estimates

Theorem

Let D be a bounded homogeneous domain in $\mathbb{C}^n$. If $\psi \in \mathcal{B}(D)$, $\varphi \in H(D, D)$, and both $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite, then $||W_{\psi,\varphi}||$ is

1. *bounded above by* $\max\{||\psi||_{\mathcal{B}}, |\psi(0)|\,\omega(\varphi(0)) + \sigma_{\psi,\varphi} + \tau_{\psi,\varphi}\}$;
2. *bounded below by* $\max\{||\psi||_{\mathcal{B}}, |\psi(0)|\,\omega(\varphi(0))\}$.

Operator Norm Estimates

Theorem

Let D be a bounded homogeneous domain in $\mathbb{C}^n$. If $\psi \in \mathcal{B}(D)$, $\varphi \in H(D, D)$, and both $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite, then $||W_{\psi,\varphi}||$ is

1. *bounded above by* $\max \{||\psi||_{\mathcal{B}}, |\psi(0)| \, \omega(\varphi(0)) + \sigma_{\psi,\varphi} + \tau_{\psi,\varphi}\}$;
2. *bounded below by* $\max \{||\psi||_{\mathcal{B}}, |\psi(0)| \, \omega(\varphi(0))\}$.

Conjecture

Let D be a bounded homogeneous domain in $\mathbb{C}^n$. If $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(D)$, then $||W_{\psi,\varphi}||$ is

1. *bounded above by* $\max \{||\psi||_{\mathcal{B}}, |\psi(0)| \, \omega(\varphi(0)) + \sigma_{\psi,\varphi} + \tau_{\psi,\varphi}\}$;
2. *bounded below by* $\max \{||\psi||_{\mathcal{B}}, |\psi(0)| \, \omega(\varphi(0))\}$.

Operator Norm Estimates

Theorem

Let D be a bounded homogeneous domain in $\mathbb{C}^n$. If $\psi \in \mathcal{B}(D)$, $\varphi \in H(D, D)$, and both $\sigma_{\psi,\varphi}$ and $\tau_{\psi,\varphi}$ are finite, then $||W_{\psi,\varphi}||$ is

1. *bounded above by* $\max\{||\psi||_{\mathcal{B}}, |\psi(0)|\,\omega(\varphi(0)) + \sigma_{\psi,\varphi} + \tau_{\psi,\varphi}\}$;
2. *bounded below by* $\max\{||\psi||_{\mathcal{B}}, |\psi(0)|\,\omega(\varphi(0))\}$.

Conjecture

Let D be a bounded homogeneous domain in $\mathbb{C}^n$. If $W_{\psi,\varphi}$ is bounded on $\mathcal{B}(D)$, then $||W_{\psi,\varphi}||$ is

1. *bounded above by* $\max\{||\psi||_{\mathcal{B}}, |\psi(0)|\,\omega(\varphi(0)) + \sigma_{\psi,\varphi} + \tau_{\psi,\varphi}\}$;
2. *bounded below by* $\max\{||\psi||_{\mathcal{B}}, |\psi(0)|\,\omega(\varphi(0))\}$.

This conjecture is true for $D = \mathbb{B}_n$.

Future Directions

- Characterize compact weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.

Future Directions

- Characterize compact weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.
- Characterize isometric weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.

Future Directions

- Characterize compact weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.
- Characterize isometric weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.
- Establish spectrum of weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.

Future Directions

- Characterize compact weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.
- Characterize isometric weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.
- Establish spectrum of weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.
- Investigate the quantities $\omega(z)$ and $B_\varphi(z)$ on bounded homogeneous domains.

Future Directions

- Characterize compact weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.
- Characterize isometric weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.
- Establish spectrum of weighted composition operators on $\mathcal{B}(\mathbb{B}_n)$.
- Investigate the quantities $\omega(z)$ and $B_\varphi(z)$ on bounded homogeneous domains.
- Prove boundedness conjecture for $D = \mathbb{D}^n$.

References

- R. Allen & F. Colonna, *On the Isometric Composition Operators on the Bloch Space in $\mathbb{C}^n$*, in submission.
- R. Allen & F. Colonna, *Characterization of Isometries and Spectra of Multiplication Operators on the Bloch Space*, in submission.
- L. Brown & A. L. Shields, *Multipliers and Cyclic Vectors in the Bloch Space*, Michigan Math. J. **38**, 1991.
- S. Ohno & R. Zhao, *Weighted Composition Operators on the Bloch Space*, Bull. Austral. Math. Soc **63**, 2001.
- R. Timoney, *Bloch Functions in Several Complex Variables, I*, Bull. London Math. Soc **12**, 1980.
- K. Zhu, *Spaces of Holomorphic Functions in the Unit Ball*, Springer-Verlag, 2004.