Mathematics 108 Name ANSWER KEY A
Quiz, Sections 3.2 & 3.4 April 24, 2009

Show all work neatly. Use of calculators is not permitted on this quiz.

1. Let $f(x)=\dfrac{12}{x^2+2}$. Find the absolute maximum and absolute minimum values of $f(x)$ on the interval $[-1, 2]$.

$f(x)=\dfrac{12}{x^2+2}$ $\quad f=12 \quad g=x^2+2$
$\quad f'=0 \quad g'=2x$

$f'(x)=\dfrac{0(x^2+2)-2x(12)}{(x^2+2)^2}=\dfrac{-24x}{(x^2+2)^2}$ 3

C.V.: let $\dfrac{-24x}{(x^2+2)^2}=0$; $-24x=0$
$\boxed{x=0}$ ← only C.V.

1

Check for max & min:
Find y-values: 2

$f(-1)=\dfrac{12}{(-1)^2+2}=\dfrac{12}{3}=4$

$\boxed{f(2)=\dfrac{12}{4+2}=\dfrac{12}{6}=2}$ min.

$\boxed{f(0)=\dfrac{12}{0+2}=\dfrac{12}{2}=6}$ max

absolute max is at the point $(0,6)$
absolute min is at the point $(2,2)$

2. The owners of an amusement park have determined that the relationship between the price p, in dollars, that is charged for admission tickets to the park and the attendance, q, on a given day is given by the function $p=60-0.03q$, for $100\le q\le 1500$.
   a) Write the revenue function R as a function of attendance, q, at the amusement park.

$R(q)=q(60-0.03q)=60q-0.03q^2$ 2

   b) For what attendance level is revenue maximized? (Be sure to find the value at which the maximum revenue occurs **and** show the results of one test for absolute maxima/minima to support the conclusion that your value is a maximum.)

maximize where $R'(q)=0=60-0.06q$

$\dfrac{6000}{6}=\dfrac{60}{.06}=\dfrac{.06q}{.06}$ 3

$\boxed{1000=q}$ (single critical value)

Note: only one C.V.
$R(q)$ is continuous 1
$R''(q)=-.06<0$, so concave down everywhere.
∴ we have an absolute maximum Revenue at an attendance level of 1000 people.
$(R(1000)=\$30{,}000)$

Mathematics 108 Name ANSWER KEY B
Quiz, Sections 3.2 & 3.4 April 24, 2009

Show all work neatly. Use of calculators is not permitted on this quiz.

1. Let $f(x) = \dfrac{6}{x^2+2}$. Find the absolute maximum and absolute minimum values of $f(x)$ on the interval $[-2, 1]$.

Find $f'(x)$:

$f = 6 \quad g = x^2+2$

$f' = 0 \quad g' = 2x$

$$f'(x) = \frac{0(x^2+2) - 2x \cdot 6}{(x^2+2)^2} = \frac{-12x}{(x^2+2)^2}$$

Let $f'(x) = 0 = \dfrac{-12x}{(x^2+2)^2}$

$-12x = 0$

$x = 0 \leftarrow$ only Critical Value

Test:

$\boxed{f(-2) = \frac{6}{(-2)^2+2} = \frac{6}{4+2} = 1}$ abs. min

$f(1) = \frac{6}{1^2+2} = \frac{6}{3} = 2$

$\boxed{f(0) = \frac{6}{0+2} = \frac{6}{2} = 3}$ abs. max

Absolute minimum at $(-2, 1)$
Absolute maximum at $(0, 3)$

2. The owners of an amusement park have determined that the relationship between the price p, in dollars, that is charged for admission tickets to the park and the attendance, q, on a given day is given by the function $p = 80 - 0.02q$, for $100 \le q \le 3500$.
   a) Write the revenue function R as a function of attendance, q, at the amusement park.

$R(q) = (80 - 0.02q)q = 80q - 0.02q^2$

   b) For what attendance level is revenue maximized? (Be sure to find the value at which the maximum revenue occurs **and** show the results of one test for absolute maxima/minima to support the conclusion that your value is a maximum.)

Find $R'(q) = 80 - 0.04q$

Let $R'(q) = 0 = 80 - 0.04q$

$\frac{0.04q}{0.04} = \frac{80}{0.04} = \frac{8000}{4} = \boxed{2000 \text{ people}}$

Do we have abs. max at $q = 2000$? Could check endpoints, but because we have a single critical value and a continuous function, we can check $R''(q) = -0.04$. $R(q)$ is concave down everywhere, so $q = 2000$ is an absolute maximum.