8 points

Math 108

Extra credit problem: 3/6/09

Let $f(x) = 2x^2 - 6x$

① Find $f'(x)$ using the limit definition.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^2 - 6(x+h) - (2x^2 - 6x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) \cancel{-6x} - 6h - 2x^2 \cancel{+6x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - 6h \cancel{-2x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 6)}{\cancel{h}} = \lim_{h \to 0} 4x + 2\overset{\nearrow 0}{\cancel{h}} - 6 = \boxed{4x - 6}$$

2. Find slope of line tangent to $f(x)$ at $x=1$.

slope $= f'(1) = 4 \cdot 1 - 6 = -2 = m_{tan}$

3. Find equation of the line tangent to $f(x)$ at $x=1$.

Point: find $f(1) = 2(1)^2 - 6(1) = 2 - 6 = -4 = y$. Point is $(1, -4)$

(both the function & tangent line go through this point)

Equation: $y - (-4) = -2(x - 1)$

or

$y + 4 = -2x + 2$

$\boxed{y = -2x - 2}$

④ Graph $f(x)$ and line tangent at $x = 1$.

$f(x) = 2x^2 - 6x = 2x(x-3)$

x-intercepts at $x = 0, x = 3$

x coord of vertex at $-\frac{(-6)}{2(2)} = \frac{6}{4} = 1\frac{1}{2}$

y-coord of vertex: $2(3/2)^2 - 6(3/2) = 2 \cdot \frac{9}{4} - 9$

$= \frac{9}{2} - 9 = -4\frac{1}{2}$