ANSWER KEY

Mathematics 108, Review questions for Chapter 4

Section 4.1 (you don't need a calculator to do any of these.)

1. Evaluate the given expressions. Your answer should be a number.
   a) $(e^2e^4)^{\frac{3}{2}} = (e^6)^{3/2} = e^{6\cdot 3/2} = \boxed{e^9}$

   b) $8^{\frac{2}{3}} + 16^{\frac{3}{4}} = (\sqrt[3]{8})^2 + (\sqrt[4]{16})^3 = (\sqrt[3]{2^3})^2 + (\sqrt[4]{2^4})^3 = 2^2 + 2^3 = 4+8 = \boxed{12}$

2. Use the properties of exponents to simplify the given expressions.
   a) $\dfrac{(x^2+y^4)^0}{(x^2y^4)^2} = \dfrac{1}{x^4y^8}$

   This is fine.

   b) $(9x^2y^{-\frac{2}{3}})^{\frac{1}{2}} = 9^{1/2}\cdot(x^2)^{1/2}\cdot(y^{-2/3})^{1/2} = 3\cdot x\cdot y^{-1/3} = \dfrac{3x}{\sqrt[3]{y}}$

3. Using the properties of exponential functions, find all real numbers x that satisfy the equations:
   rewrite with a common base.
   a) $3^{5x-2} = 9^x = (3^2)^x = 3^{2x}$; $5x-2 = 2x$; $-2 = -3x$; $\boxed{x = 2/3}$
   set exponents equal to each other

   isolate exponential term! b) $7 = 2 + 5e^{-6x}$; $-2 \quad -2$; $\dfrac{5}{5} = \dfrac{5\cdot e^{-6x}}{5}$; $1 = e^{-6x}$; but $1 = e^0$, so $e^0 = e^{-6x}$; $0 = -6x$, $\boxed{x = 0.}$

   c) $400 = 100 + 3\cdot 10^{-5x}$; $-100 \quad -100$; $\dfrac{300}{3} = \dfrac{3\cdot 10^{-5x}}{3}$; $100 = 10^{-5x}$; but $100 = 10^2$, so $10^2 = 10^{-5x}$, $2 = -5x$, $\boxed{x = -2/5.}$

Section 4.3

Differentiate the following functions:

Quotient Rule → 1. $f(x) = \dfrac{x^2+4x+1}{e^x}$. $\boxed{f'(x) = \dfrac{(2x+4)e^x - (x^2+4x+1)e^x}{(e^x)^2}}$ $\quad f = x^2+4x+1$, $f' = 2x+4$; $g = e^x$, $g' = e^x$

Product Rule 2. $f(x) = (x^2-3x+9)(e^{3x})$
   $f'(x) = (2x-3)e^{3x} + (x^2-3x+9)(3e^{3x})$ $\quad f = x^2-3x+9$, $f' = 2x-3$; $g = e^{3x}$, $g' = e^{3x}\cdot 3 = 3e^{3x}$

Chain Rule 3. $f(x) = e^{(x^3-4x)}$ $\quad \boxed{f'(x) = e^{x^3-4x}(3x^2-4)}$

Product Rule 4. $y = x^3e^{(x^2-6x)}$
   $y' = 3x^2(e^{x^2-6x}) + x^3(e^{x^2-6x})(2x-6)$ $\quad f = x^3$, $f' = 3x^2$; $g = e^{x^2-6x}$, $g' = e^{x^2-6x}(2x-6)$ (chain Rule)

Word Problems

1. It is determined that q units of a commodity can be sold when the price is p hundred dollars per unit, where q is a function of p, and $q(p) = 2000(p)e^{-p}$.
   a) Find the Revenue function: $R(p) = p\cdot q(p) = p\cdot(2000\,p\cdot e^{-p}) = \boxed{2000p^2e^{-p}.}$

   q is a function of p

$f = 2000p^2 \quad g = e^{-p}$

$f' = 4000p \quad g' = e^{-p}(-1)$

use Product Rule

b) Find the marginal revenue function: $R'(p) = 4000p \cdot e^{-p} - 2000p^2 \cdot e^{-p}$

c) At what value of p is Revenue maximized?

Let $R'(p) = 0 = \underbrace{2000e^{-p}}_{\text{never equal 0}}(2-p)$

Let $2 - p = 0$

$p = 2$ (represents a price of \$200)

d) What is the maximum revenue? (would require a calculator to compute)

Maximum revenue $= R(2) = 2000 \cdot 2^2 \cdot e^{-2} = \dfrac{2000 \cdot 4}{e^2} = \dfrac{\$8000}{e^2} \doteq \$1082.62$ (hundred)

as far as you can get without a calculator

2. It is determined that q units of a commodity can be sold when the price is p dollars per unit, where q is a function of p, and $q(p) = 5000pe^{-2p}$. The revenue obtained from the sale of q units at price p is $R(p) = p \cdot q(p)$. $= p(5000)p \cdot e^{-2p} = 5000 \cdot p^2 \cdot e^{-2p}$

a) Find the marginal revenue $R'(p)$: (use Product Rule)

$f = 5000p^2 \quad g = e^{-2p}$

$f' = 10000p \quad g' = e^{-2p}(-2) = -2e^{-2p}$

$R'(p) = 10000p(e^{-2p}) - 2 \cdot e^{-2p}(5000p^2)$

$= 10000p(e^{-2p}) - 10000p^2 \cdot e^{-2p}$

b) Find all critical values of the function $R(p)$.

Let $10000p(e^{-2p}) + 10000p^2e^{-2p} = 0$

$10000pe^{-2p}(1-p) = 0$

$\underbrace{10000e^{-2p}}_{\text{Never} = 0} \neq 0 \quad p = 0 \quad 1 - p = 0$

$p = 1$ $\boxed{\text{CV's are } p = 0, p = 1}$

c) At what price p is revenue maximized?

Maximized at $p = 1$ (relative max, only max in interval $(0, +\infty)$).

+ everywhere →

$10000e^{-2p}$	+	+	+
p	−	+	+
$1-p$	+	+	−
	−	+	−

0 1

Can get here without calculator

d) What is the maximum revenue? $\boxed{\dfrac{\$5000}{e^2}} \doteq \$676.68$

$R(1) = 5000 \cdot 1^2 \cdot e^{-2(1)} =$

3. Suppose Fred and Judy have \$10,000 to invest for a period of 5 years. They can put their \$10,000 in an account at Bob's Bank which pays 3% per year, compounded monthly, or in an account at Bank of the Universe, which pays 2.9% per year, compounded continuously. Compute the amount they would have at the end of 5 years with both investment choices. Which is the better investment? ___________

(Requires a calculator)

Bob's Bank: $B(t) = 10{,}000\left(1+\frac{.03}{12}\right)^{12\cdot 5} = \$11{,}616.167\ldots \approx \boxed{\$11{,}616.17}$

Bank of the Universe: $B'(t) = 10{,}000 \cdot e^{.029(5)} = \$11{,}560.395\ldots \approx \boxed{\$11{,}560.40}$

Better investment is Bob's Bank

4. How much money should be invested now at 5% to obtain \$10,000 in 6 years, if the interest is compounded each of the following 2 ways:

a) Monthly $n=12$

b) Continuously

If $B(t) = P\left(1+\frac{r}{n}\right)^{nt}$, then $P = \frac{B(t)}{\left(1+\frac{r}{n}\right)^{nt}}$

a) $P = \frac{10{,}000}{\left(1+\frac{.05}{12}\right)^{12\cdot 6}} = \$7412.8009\ldots \approx \boxed{\$7412.81}$

b) Continuously: if $B(t) = P \cdot e^{rt}$, then $P = \frac{B(t)}{e^{rt}} = \frac{10000}{e^{.05\times 6}} = 7408.182$

$\boxed{\$7408.19}$