Math 108, Business Calculus
Quiz, Implicit Differentiation

Name ANSWER KEY A
April 10, 2009

Please show all work neatly. Use of calculators is not permitted.

6 pts.

1. Find the derivative y' or $\frac{dy}{dx}$, where $xy + 5y = x^4$. You may use implicit differentiation or you may solve by differentiating an explicit formula for y.

xy is a product

$f = x \quad g = y$
$f' = 1 \quad g' = y'$

$y + xy' + 5y' = 4x^3$

$xy' + 5y' = 4x^3 - y$

$y'(x+5) = 4x^3 - y$

$\boxed{y' = \frac{4x^3 - y}{x+5}}$

or: $y(x+5) = x^4$

$y = \frac{x^4}{x+5}$

$f = x^4 \quad g = x+5$
$f' = 4x^3 \quad g' = 1$

or $\boxed{y' = \frac{4x^3(x+5) - x^4}{(x+5)^2}}$

2. Suppose that $x^2(x+y) + y^5 = 3$.

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a) Find the derivative y' or $\frac{dy}{dx}$ using the equation above. You must use implicit differentiation to solve this problem (doing algebra first is always allowed.)

$x^3 + x^2y + y^5 = 3$ (algebra first)

$f = x^2 \quad g = y$
$f' = 2x \quad g' = y'$

$3x^2 + 2xy + x^2y' + 5y^4 \cdot y' = 0$

$-3x^2 - 2xy \qquad -3x^2 - 2xy$

$x^2y' + 5y^4y' = -3x^2 - 2xy$

$y'(x^2 + 5y^4) = -3x^2 - 2xy$

$\boxed{y' = \frac{-3x^2 - 2xy}{x^2 + 5y^4}}$

OR (product rule)

$f = x^2 \quad g = x + y$
$f' = 2x \quad g' = 1 + y'$

$2x(x+y) + x^2(1+y') + 5y^4 \cdot y' = 0$

$-2x(x+y) \qquad -2x(x+y)$

$x^2 + x^2y' + 5y^4y' = -2x(x+y)$

$-x^2 \qquad -x^2$

$x^2y' + 5y^4y' = y'(x^2+5y^4) = -2x(x+y) - x^2$

$y' = \frac{-2x(x+y) - x^2}{x^2 + 5y^4}$

b) Find an equation of the line tangent to the curve above at the point (1, 1).

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slope $= m_{tan} = \frac{-3(1)^2 - 2(1)(1)}{1^2 + 5(1^4)} = \frac{-3-2}{1+5} = \frac{-5}{6}$

equation: $y - 1 = -5/6\,(x-1)$

or

$= -\frac{5}{6}x + \frac{5}{6}$

$y = -\frac{5}{6}x + \frac{11}{6}$

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Name ANSWER KEY B
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Please show all work neatly. Use of calculators is not permitted.

1. Find the derivative y' or $\frac{dy}{dx}$, where $xy + 4y = x^3$. You may use implicit differentiation or you may solve by differentiating an explicit formula for y.

xy is a product

$f = x \quad g = y$
$f' = 1 \quad g' = y'$

$y + xy' + 4y' = 3x^2$
$xy' + 4y' = 3x^2 - y = y'(x+4)$
$\boxed{y' = \frac{3x^2 - y}{x+4}}$

OR $y(x+4) = x^3$
$y = \frac{x^3}{x+4}$

$f = x^3 \quad g = x+4$
$f' = 3x^2 \quad g' = 1$

$y' = \frac{3x^2(x+4) - x^3}{(x+4)^2}$

2. Suppose that $x^3(x+y) + y^4 = 3$.
   a) Find the derivative y' or $\frac{dy}{dx}$ using the equation above. You must use implicit differentiation to solve this problem (doing algebra first is always allowed.)

$x^4 + x^3y + y^4 = 3$

$f = x^3 \quad g = y$
$f' = 3x^2 \quad g' = y'$

$4x^3 + 3x^2y + x^3y' + 4y^3y' = 0$
$x^3y' + 4y^3y' = -4x^3 - 3x^2y = y'($
$y' = \frac{-4x^3 - 3x^2y}{x^3 + 4y^3}$

OR: (product rule)

$f = x^3 \quad g = x+y$
$f' = 3x^2 \quad g' = 1 + y'$

$3x^2(x+y) + x^3(1+y') + 4y^3 \cdot y' = 0$
$x^3 + x^3y' + 4y^3y' = -3x^2(x+y)$
$y'(x^3 + 4y^3) = -3x^2(x+y) - x^3$; $y' = \frac{-3x^2(x+y) - x^3}{x^3 + 4y^3}$

   b) Find an equation of the line tangent to the curve above at the point (1, 1).

at $(1,1)$, $m_{tan} = \frac{-4(1) - 3(1)}{1+4} = \frac{-7}{5}$

Equation: $y - 1 = -7/5(x-1)$ or

$= -7/5x + 7/5$

$y = -7/5x + 12/5$