Math 108 Final Exam Topics, Spring 2009 ANSWER KEY

1. Domain

- $g(x) = \frac{12x}{x^2-4x-5}$ Rational function: Issue - are there values of x where denominator = 0?

Let $x^2-4x-5=0$

$(x-5)(x+1)=0$

$x=5 \quad x=-1$ ∴ Exclude these values from domain.

$\boxed{\text{Domain: } \{x \mid \neq 5, x \neq -1\}}$

- $f(x)=\sqrt{9-x}$. Square root: Issue - quantity under radical must be ≥ 0.

Let $9-x \geq 0$

$9 \geq x$, or $x \leq 9$.

$\boxed{\text{Domain: } \{x \mid x \leq 9\}}$

- $f(x) = \frac{3x+6}{\sqrt{x^2-1}}$ Square root in denominator: quantity under radical must be >0 (can't equal 0, because it's in the denominator)

solution: $(x^2-1)>0$

$(x+1)(x-1)>0$

let $(x+1)(x-1)=0$

$x=-1 \quad x=1$

Check sign

-2 $\quad$ 0 $\quad$ 2

$4-1=3$ $\quad$ $0-1=-1$ $\quad$ $4-1=3$

$+ \quad -1 \quad - \quad 1 \quad +$

$\boxed{\begin{array}{l}\text{Domain (where } x^2-1>0 \text{ or } x^2-1 \text{ is "+")} \\ \{x \mid x<-1 \text{ or } x>1\}\end{array}}$

(or $(-\infty,-1) \cup (1,+\infty)$)

- $g(x)=x^4-6x^3+5x-3$. Polynomial - domain is all reals.

$\boxed{\text{Domain: } \{x \mid x \in \mathbb{R}\}}$

- $g(x)=e^{4x}$. Exponential function - domain is all real numbers.

$\boxed{\text{Domain: } \{x \mid x \in \mathbb{R}\}}$

- $f(x)=\frac{x^2-4}{2^x}$. Issue: variable in denominator. Q: Can 2^x ever equal 0? No. $\lim_{x\to-\infty} 2^x=0$, but $2^x \neq 0$. ∴ No problem in the denominator, and domain is all real numbers. $\boxed{\text{Domain: } \{x \mid x \in \mathbb{R}\}}$

2. $f(x)=\frac{x-1}{x^2-5x+4}$

$\boxed{\lim_{x\to2} f(x) = -\frac{1}{2}}$

plug in: $\frac{2-1}{4-10+4} = \frac{1}{-2}$

$\boxed{\lim_{x\to1} f(x) = \lim_{x\to1} \frac{\overset{1}{\cancel{x-1}}}{(\cancel{x-1})(x-4)} = \frac{1}{-3} = \frac{-1}{3}}$

plug in: $\frac{1-1}{1-5+4} = \frac{0}{0}$: keep going (indeterminate)

$\boxed{\lim_{x\to4} f(x) = \text{DNE (Vertical Asymptote)}}$

plug in $\frac{4-1}{16-20+4} = \frac{3}{0}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\frac{x}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}-\frac{5x}{x^2}+\frac{4}{x^2}} = \lim_{x\to\infty} \frac{\overset{\nearrow 0}{\cancel{\frac{1}{x}}} - \overset{\nearrow 0}{\cancel{\frac{1}{x^2}}}}{1-\underset{\downarrow 0}{\frac{5}{x}}+\underset{\downarrow 0}{\frac{4}{x^2}}} = \frac{0}{1} = \boxed{0}$$

↑ Horizontal Asymptote

3. at $x=2$: Check

$$\lim_{x\to 2^-} f(x) = 2^3 = 8$$

$$\lim_{x\to 2^+} f(x) = 4\cdot 2 - 1 = 7$$

$$\therefore \lim_{x\to 2} f(x) \text{ DNE} \quad \left(\text{because } \lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)\right)$$

$$\boxed{\therefore f(x) \text{ is not continuous at } x=2.}$$

at $x=3$, $f(x) = 4x-1$ (a line). The function $4x-1$ is continuous everywhere $\therefore$ $f(x)$ is continuous at $x=3$.

4. Definition. Given a function $f(x)$, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, where the limit exists.

- find $f'(x)$, where $f(x) = 2x^2 - 3x$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 - 3(x+h) - (2x^2-3x)}{h} = \lim_{h\to 0} \frac{2(x^2+2xh+h^2) \cancel{-3x} - 3h - 2x^2 \cancel{+3x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - 3h - \cancel{2x^2}}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x + \cancelto{0}{2h} - 3)}{\cancel{h}} = \boxed{4x-3}$$

5. a) $f(x) = 4x^5 + 12x^4 + (x^4)^{1/3} + \pi^3 = 4x^5 + 12x^4 + x^{4/3} + \underset{\substack{\uparrow \\ \text{constant!}}}{\pi^3}$

$$\boxed{f'(x) = 20x^4 + 48x^3 + 4/3x^{1/3} + 0}$$

b) $f(x) = \frac{3x^2+1}{5x+2}$ (Quotient Rule)

$f = 3x^2+1 \nearrow g = 5x+2$

$f' = 6x \qquad g' = 5$

Does NOT cancel!

$$f'(x) = \underbrace{\frac{(6x)(5x+2) - 5(3x^2+1)}{(5x+2)^2}}_{\text{fine to leave like this.}} = \frac{30x^2+12x-15x^2-5}{(5x+2)^2} = \frac{15x^2+12x-5}{(5x+2)^2}$$

c) $f(x) = \frac{3}{(\sqrt{9x^3}+2x)^4} = 3\left((9x^3)^{1/2} + 2x\right)^{-4}$ (rewrite, to avoid quotient rule)

$$f'(x) = -12\left((9x^3)^{1/2} + 2x\right)^{-5} \underbrace{\left(\overset{\substack{\text{chain rule} \\ \downarrow}}{1/2 (9x^3)^{-1/2} (27x^2)} + 2\right)}_{\text{Chain rule}}$$

OK, with Quotient Rule

$$f'(x) = \frac{-3(4)(3x^{3/2}+2x)^3(9/2\, x^{1/2}+2)}{(3x^{3/2}+2x)^8}$$

$f = 3 \qquad g = (\sqrt{9x^3}+2x)^4 = \overset{\substack{\text{okay to rewrite like} \\ \text{this, but do it} \\ \text{carefully!} \\ \downarrow}}{(3x^{3/2}+2x)^4}$

$f' = 0 \qquad g' = 4(3x^{3/2}+2x)^3(\frac{9}{2}x^{1/2}+2)$

d) $f(x) = (3x^2-7)^5\left(\frac{2}{3x}+5\right)^2 = (3x^2-7)^5\left(\frac{2}{5}\cdot x^{-1}+5\right)^2$

Use product Rule

$f=(3x^2-7)^5 \quad g=\left(\frac{2}{5}\cdot x^{-1}+5\right)^2$

$f' = 5(3x^2-7)^4(6x) \quad g' = 2\left(\frac{2}{5}x^{-1}+5\right)\left(-\frac{2}{5}x^{-2}\right)$

↑ Chain Rule ↑ Chain Rule

$$\boxed{f'(x) = \underbrace{30x(3x^2-7)^4\left(\frac{2}{5x}+5\right)^2 + (3x^2-7)^5\left(\frac{-4}{5x^2}\right)\left(\frac{2}{5x}+5\right)}_{\text{fine to leave like this, or to leave negative exponents}}}$$

e) $f(x) = \left(\underbrace{\sqrt[4]{x^2+1}}\right)\left(\underbrace{e}^{3x^2-4}\right)$

product Rule

$f=(x^2+1)^{1/4} \quad g=e^{3x^2-4}$

$f' = 1/4(x^2+1)^{-3/4}(2x) \quad g' = e^{3x^2-4}\cdot 6x$

$= \frac{x}{2}(x^2+1)^{-3/4}$

$$\boxed{f'(x) = \frac{x}{2}(x^2+1)^{-3/4}\cdot e^{3x^2-4} + (x^2+1)^{1/4}\left(e^{3x^2-4}\right)(6x)}$$

f) $f(x) = (x^2-3x+9)(\sqrt{3x})$

Use Product Rule

$f = x^2-3x+9 \quad g = (3x)^{1/2}$

$f' = 2x-3 \quad g' = 1/2(3x)^{-1/2}\cdot 3$

$= 3/2(3x)^{-1/2}$

$$\boxed{f'(x) = (2x-3)(\sqrt{3x}) + (x^2-3x+9)(3/2)(3x)^{-1/2}}$$

g) $f(x) = \underbrace{(x^3-e^{5x})}(x+7)^3$

Product: Use Product Rule

$f = (x^3-e^{5x}) \quad g = (x+7)^3$

$f' = 3x^2 - \underbrace{e^{5x}\cdot 5}_{\text{exponential}} \quad g' = 3(x+7)^2\cdot 1$

$$\boxed{f'(x) = (3x^2-5e^{5x})\cdot(x+7)^3 + 3(x+7)^2(x^3-e^{5x})}$$

h) $f(x) = e^{4x^2-7x}$ $\quad \boxed{f'(x) = \left(e^{4x^2-7x}\right)(8x-7)}$

↑ chain Rule

6. Implicit:

$xy - x^4 + 4 = y^3$

↑ product

$f = x \quad g = y$

$f' = 1 \quad g' = y'$

Chain rule ↓

$y + xy' - 4x^3 + 0 = 3y^2\cdot y'$

Isolate y' terms: $xy' - 3y^2y' = 4x^3 - y$

factor out y' : $y'(x-3y^2) = 4x^3-y$

Divide: $\boxed{y' = \frac{4x^3-y}{x-3y^2}} = \frac{y-4x^3}{3y^2-x}$

Final Review: Page 2

Second derivatives

7. • $f(x) = (6x^3-2x)^4$

$f'(x) = 4(6x^3-2x)^3(\underset{\uparrow}{18x^2-2})$

chain Rule

use product rule to find 2nd derivative ↓

$f = 4(6x^3-2x)^3 \quad g = 18x^2-2$

$f' = 12(6x^3-2x)^2(18x^2-2) \nearrow g' = 36x$

$$\boxed{\underbrace{f''(x) = 12(6x^3-2x)^2(18x^2-2)^2 + 4(6x^3-2x)^3(36x)}_{\text{Fine to leave like this}}}$$

(needs Product Rule)

• $f(x) = 5000x^2e^{-2x^2}$ (Need Product rule) →

$f = 5000x^2 \quad g = e^{-2x^2}$

$f' = 10000x \nearrow g' = e^{-2x^2}\cdot(-4x)$

$f' = 10000x(e^{-2x^2}) + 5000x^2(e^{-2x^2})(-4x)$

$= 10000x(e^{-2x^2}) - 20000x^3(e^{-2x^2})$

$= e^{-2x^2}(10000x - 20000x^3)$ ← Need product rule for 2nd derivative

$$\boxed{f''(x) = e^{-2x^2}(-4x)(10000x - 20000x^3) + (e^{-2x^2})(10000 - 60000x^2)}$$

(use product rule)

$f = e^{-2x^2} \quad g = 10000x - 20000x^3$

$f' = e^{-2x^2}(-4x) \nearrow g' = 10000 - 60000x^2$

8. Equations of tangent line:

a) $f(x) = (2x^3-1)^2$ at $x=1$

i. Find point: $f(1) = (2\cdot1^3-1)^2 = (1)^2 = 1$

(1, 1) point

ii) $f'(x) = 2(2x^3-1)^1(6x^2) = 12x^2(2x^3-1)$

iii) find slope by finding $f'(1) = 12\cdot1^2(2\cdot1^3-1) = 12(2-1) = 12 = m_{tan}$

iv) $\boxed{\text{Equation of line: } y-1 = 12(x-1)}$

or $y-1 = 12x-12$

$\boxed{y = 12x-11}$

b) $f(x) = \frac{3x^4+2x}{5}$ at $x=1$ i) point: $f(1) = \frac{3(1)^4+2(1)}{5} = \frac{5}{5} = 1$

point: $(1,1)$

ii) $f'(x) = \frac{12x^3+2}{5}$ (or $1/5 \cdot 12x^3+2$)

iii) $f'(1) = \frac{12 \cdot 1+2}{5} = \frac{14}{5}$ iv. line: $y-1 = \frac{14}{5}(x-1)$ or $\boxed{y-1 = \frac{14}{5}x - \frac{14}{5} \quad y = \frac{14}{5}x - \frac{9}{5}}$

c) $f(x) = e^{2x^3-2}$ at $x=1$ i) point: $f(1) = e^{2 \cdot 1-2} = e^0 = 1$

point: $(1,1)$

ii) $f'(x) = e^{2x^3-2} \cdot 6x$

iii) $f'(1) = e^{2 \cdot 1-2} \cdot 6 \cdot 1 = e^0 \cdot 6 = 6 = m_{tan}$ iv) line: $\boxed{y-1 = 6(x-1)}$

OR

$y-1 = 6x-6$

$\boxed{y = 6x-5}$

9. a) $f(x) = 3x^4 - 8x^3 + 16$

$f'(x) = 12x^3 - 24x^2$

$= 12x^2(x-2)$

let $f'(x) = 0$, then $\underbrace{x=0, \ x=2}_{\text{critical values}}$

	-1		1		3
x^2	+		+		+
$(x-2)$	-		-		+
	-	0	-	2	+
	↘		↘		↗

Interval of increase: $(2, +\infty)$

Intervals of decrease: $(-\infty, 0), (0, 2)$

Classify: $x=0$ is neither a maximum nor minimum (no change in direction)

$x=2$ is a relative minimum

2nd derivative test: $f''(x) = 36x^2 - 48x$

$f''(0) = 0$: inconclusive (have to use 1st derivatives)

$f''(2) = 144 - 96 = 48$ ∪ 2 is a min ✓

b) $g(x) = \frac{3x+5}{x^2-1}$. $g'(x) = \frac{3(x^2-1) - 2x(3x+5)}{(x^2-1)^2} = \frac{3x^2-3-6x^2-10x}{(x^2-1)^2} = \frac{-3x^2-10x-3}{(x^2-1)^2}$

$= \frac{-1(3x^2+10x+3)}{(x^2-1)^2} = \frac{-1(3x+1)(x+3)}{(x^2-1)^2}$ To find CV's let $-1(3x+1)(x+3) = 0$

$\boxed{x = -1/3 \quad x = -3}$

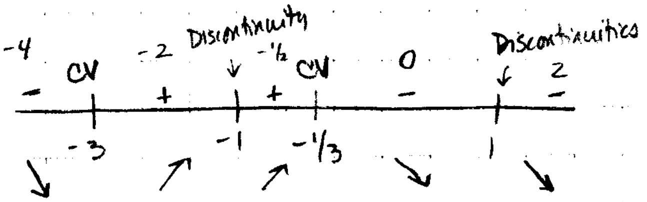
To find discontinuities, let $(x^2-1) = 0$

$x = \pm 1$

Check intervals:

$g' \frac{-1(3x^2+10x+3)}{(x^2-1)^2}$

(NOTE: Don't lose negative in numerator)



← Have to check ALL of these, because the function may change direction at any of them, including discontinuities.

Intervals of increase: $(-3,-1)\cup(-1,-1/3)$

Intervals of decrease: $(-\infty,-3)\cup(-1/3,1)\cup(1,+\infty)$

(note: I omitted points of discontinuity because they are not in the domain of g.)

$x=-3$: relative minimum

$x=-1/3$: relative maximum

2nd derivative test (The final will NOT have problems this messy!)

$$g''(x)=\frac{(-6x-10)(x^2-1)^2-(-3x^2-10x-3)(4x)(x^2-1)}{(x^2-1)^4}$$

$f=-3x^2-10x-3$ $g=(x^2-1)^2$

$f'=-6x-10$ $g'=2(x^2-1)(2x)$

$$=\frac{-(6x+10)(x^2-1)+(3x^2+10x+3)(4x)}{(x^2-1)^3}$$

$$g''(-3)=\frac{\overset{(-6)(-8)(8)}{-6(-18+10)(9-1)}+(3\cdot 9\overset{0}{\cancel{-30+3}})(4)(-3)}{(9-1)^3}\quad + \quad \cup \quad \text{rel. minimum at } x=-3\ ✓$$

$$g''(-1/3)=\frac{-(-2+10)(\overset{8}{\frac{1}{9}}-1)+0}{(\frac{1}{9}-1)^3}\overset{3}{} = \frac{-[(8)(-8/9)](+)}{(-8/9)^3\ (-)} \quad \frac{+}{-}\ - \quad \cap \quad \text{rel. maximum at } x=-1/3$$

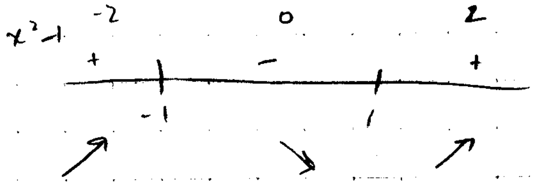
- $f(x)=e^{x^3-3x}$

$f'(x)=e^{x^3-3x}(3x^2-3)=e^{x^3-3x}(3)(x^2-1)$

let $\underbrace{e^{x^3-3x}}_{\text{Never}=0,\ \text{and always positive}}(3)(x^2-1)=0$

$(x-1)(x+1)=0$

$x=1\quad x=-1$



Intervals of increase: $(-\infty,-1)\cup(1,+\infty)$

Intervals of decrease: $(-1,1)$

$x=-1$: relative max

$x=1$ relative min.

2nd derivative test:

$f=3e^{x^3-3x}$ $g=x^2-1$

$f'=3\cdot e^{x^3-3x}(x^2-1)$, $g'=2x$

$f''(x)=3e^{x^3-3x}(x^2-1)^2+2x(3e^{x^3-3x})$

$=3e^{x^3-2x}[(x^2-1)^2+2x]$

$f''(1)=3e^{1-2}[(1-1)+2]=3\cdot e^{-1}\cdot 2\quad + \quad \cup \Rightarrow$

$x=1$ is a rel. minimum ✓

$f''(-1)=3e^{(-1)^3+2}[((-1)^2-1)^2-2]=3e^1(0-2)\ - \quad \cap$

$x=-1$ is a relative maximum ✓

- $g(x) = (2x^2+1)e^x$
  need product Rule

$f = 2x^2+1 \qquad g = e^x$

$f' = 4x \qquad g' = e^x$

$g'(x) = 4x \cdot e^x + (2x^2+1)(e^x)$

$= e^x(2x^2+1+4x)$

Let $e^x(2x^2+4x+1) = 0$

$\underbrace{e^x \neq 0}_{\text{never}=0} \qquad 2x^2+4x+1 = 0$

Use quadratic formula:

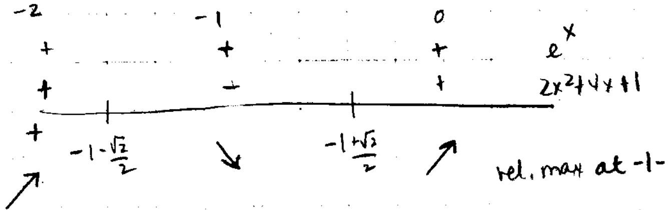
$x = \frac{-4 \pm \sqrt{16-4\cdot2\cdot1}}{2\cdot2} = \frac{-4\pm\sqrt{8}}{4} = \frac{-4\pm2\sqrt{2}}{4} = \frac{-2\pm\sqrt{2}}{2} = -1 \pm \frac{\sqrt{2}}{2}$

2nd derivative: $(4x+4)(e^x) + e^x(2x^2+4x+1)$

$= \overset{\overset{+}{\uparrow}}{e^x}(\underbrace{2x^2}\underbrace{+8x+5})$

would really need a calculator here (just too messy - a limitation of 2nd derivative test)

CV's: $\frac{-2\pm\sqrt{2}}{2} = -1 \pm \frac{\sqrt{2}}{2}$



rel. max at $-1-\frac{\sqrt{2}}{2}$

rel min at $-1+\frac{\sqrt{2}}{2}$

Int of increase: $(-\infty, -1-\frac{\sqrt{2}}{2}), (-1+\frac{\sqrt{2}}{2}, +\infty)$

Int of decrease $(-1-\frac{\sqrt{2}}{2}, 1-\frac{\sqrt{2}}{2})$

10. b) $(\pi^2\sqrt{\pi})^{2/3} = (\pi^2\pi^{1/2})^{2/3}$

$= (\pi^{5/2})^{2/3} = \pi^{10/6} = \boxed{\pi^{5/3}}$

a) $\frac{x^4+x^6}{(x^3y^2)^5} = \frac{x^4(1+x^2)}{x^{15}y^{10}} = \boxed{\frac{1+x^2}{x^{11}y^{10}}}$ (can't simplify further)

12. a) $10 = 2 + \frac{1}{2}\cdot 4^{3x}$ (step 1: isolate exponential term)

$\quad -2 \quad -2$

$2 \cdot 8 = \frac{1}{2}\cdot 4^{3x} \cdot 2$ continue to isolate 4^{3x}

$16 = 4^{3x}$

$4^2 = 4^{3x} \Rightarrow 2 = 3x \Rightarrow \boxed{x = 2/3}$

- express 16 as a power of 4, so bases are equal
- set exponents equal to each other.

12. a) Marginal cost = $C'(x) = 1/8 \cdot 2q + 1 = 1/4 q + 1$

b) Revenue function: $q \cdot (23 - q) = 23q - q^2 = R(q)$

Marginal revenue: $23 - 2q = R'(q)$

c) To estimate cost of 5th unit, find $C'(\overset{\wedge}{4}) = 1/4 \cdot 4 + 1 = \$2.00$

drop down 1

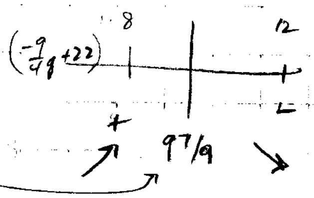
d) $P(q) = R(q) - C(q) = 23q - q^2 - (1/8 q^2 + q + 20) = 23q - q^2 - 1/8 q^2 - q - 20$
$= -9/8 q^2 + 22q - 20$

e) Marginal profit: $P'(q) = -\frac{9}{4}q + 22$

f) Where is profit maximized? let $-\frac{9}{4}q + 22 = 0$, $22 = 9/4 q$

$q = 22 \cdot \frac{4}{9} = \frac{88}{9} = 9 7/9 \sim 10$ units

Is it a max? yes



14. # sold $= 1000 \cdot e^{-0.02p} = q$

cost per unit to make: \$125

cost per unit to sell: p

Revenue: $\underset{(R(p))}{p} \cdot (\underbrace{1000\, e^{-0.02p}}_{q})$ Cost: $\underset{(C(p))}{\overset{\text{cost per unit}}{\overset{\downarrow}{125}}} \, (\overset{\text{\# sold}}{\underbrace{1000 e^{-0.02p}}_{q}}) \leftarrow$ Total cost

a) Profit $= P(p) = \underset{\text{Revenue}}{R(p)} - \underset{\text{Cost}}{C(p)} = 1000\, p \cdot e^{-0.02p} - 125\,(1000\, e^{-0.02p}) = 1000\, e^{-0.02p}(p-125)$

b) to maximize profit, find $P'(p)$ use product rule on 1st term. $f = 1000 e^{-0.02p}$ $g = p-125$

$f' = 1000 e^{-0.02p}(-0.02)$ $g' = 1$

$$P'(p) = 1000\, e^{-0.02p}(-0.02)(p-125) + 1000\, e^{-0.02p}$$
$$= 1000\, e^{-0.02p}[(-0.02)(p-125)+1] =$$
$$1000\, e^{-0.02p}[-0.02p + 2.5 + 1] = 1000\,(e^{-0.02p})(3.5-0.02p)$$

Let $P'(p) = 0$; $\underbrace{1000\, e^{-0.02p}}_{\text{Never} = 0}(3.5 - 0.02p)$.

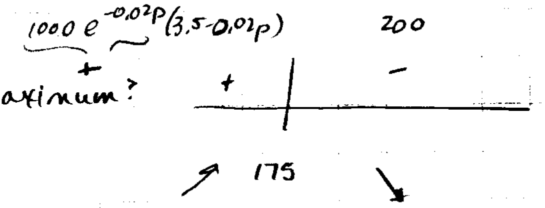
$3.5 - 0.02p = 0$

$3.5 = 0.02p$

$p = 175$ (price per unit, so $p = \$175$)

∴ Manufacturer should sell at \$175 per unit to maximize profit.

Is it a maximum? yes



14. a $g(p) = 5000\, p \cdot e^{-2p}$

Revenue $= p \cdot g(p) = p \cdot 5000 \cdot p \cdot e^{-2p}$

$= p^2 \cdot 5000 \cdot e^{-2p}$

a) Marginal revenue $= R'(p)$

$f = 5000p^2$ $g = e^{-2p}$

$f' = 10000p$ $g' = e^{-2p}(-2)$

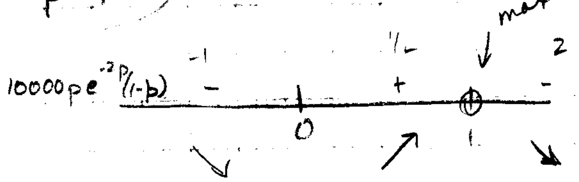
$$R'(p) = 10000p\,(e^{-2p}) + 5000p^2\,(e^{-2p})(-2)$$
$$= 10000p\,(e^{-2p}) - 10000p^2 e^{-2p}$$
$$= 10000\, p\,(e^{-2p})(1-p)$$

b) Let $R'(p) = 0 = 10000\overset{\text{never}=0}{p}\,(\overset{\text{never}=0}{e^{-2p}})(1-p)$

$p = 0$ $1-p = 0$

$p = 1$ critical values

c) $\boxed{\text{Maximum revenue at } p = \$1}$



maximum at $p = 1$

d) Maximum revenue is $R(1) =$

$1^2(5000) \cdot e^{-2 \cdot 1} = \$\frac{5000}{e^2}$ ($\sim$ \$677) if $B(t) = P(1+\frac{r}{n})^{nt}$, then $P = \frac{B}{(1+\frac{r}{n})^{nt}}$

16. a) (Present Value problem)

i) $10000 = P(1+\frac{.07}{12})^{12 \cdot 6}$ $n = 12$ (monthly)

$r = .07$

$P = \frac{10000}{(1+\frac{.07}{12})^{72}} \overset{\sim}{=} \$6578.49$

ii) $P = \frac{10000}{(1+\frac{.07}{4})^{4 \cdot 6}} =$ $n = 4$ (quarterly)

$\frac{10000}{(1+\frac{.07}{4})^{24}} \overset{\sim}{=} \$6594.38$

iii) $P = \frac{10000}{e^{.07 \cdot 6}} \overset{\sim}{=} \$6570.47$ If $B(t) = Pe^{rt}$, then $P = \frac{B}{e^{rt}}$

~~b) How long? $B = P \cdot e^{rt}$ (want B to quadruple, so it will be $4 \cdot 4500 = \$18000$)~~

Not responsible for!

~~$\frac{18000}{4500} = \frac{4500 \cdot e^{.045t}}{4500}$ – isolate exponential term~~

Did not cover.

~~$4 = e^{.045t}$ – take ln of both sides~~

~~$\ln 4 = \ln e^{.045t} = .045t$~~

~~$t = \frac{\ln 4}{.045}$ } answer w/out calculator Divide by .045~~

~~$\overset{\sim}{=} 30.8$ years~~

c) Future value:

a) annually: $n=1$ $B(t) = 1000(1+.032)^5 \overset{\sim}{=} \$1170.57$

b) semiannually: $n=2$ $B(t) = 1000(1+\frac{.032}{2})^{2 \times 5} \overset{\sim}{=} \$1172.03$

c) quarterly: $n=4$ $B(t) = 1000(1+\frac{.032}{4})^{4 \times 5} \overset{\sim}{=} \$1172.76$

d) monthly: $n=12$ $B(t) = 1000(1+\frac{.032}{12})^{12 \times 5} \overset{\sim}{=} \$1173.26$

e) continuously: $B(t) = 1000 \cdot e^{(.032) \times 5} \overset{\sim}{=} \$1173.51$

17. <u>Population:</u> $P = 50 \cdot e^{.004t}$

a) Current population = $P(0) = 50 \cdot e^{.004 \cdot 0} = 50 \cdot 1 =$ 50 thousand people, or
50,000 people

b) Population in 20 years: $P(20) = 50 \cdot e^{.004 \times 20} \doteq$ 54.164 thousand people or
56,164 people

c) $\underline{\underline{\text{Rate}}}$ at which population will be changing 20 yrs from now:
↑
derivative

$P'(t) = 50 \cdot e^{.004t}(.004) = .2e^{.004t}$

In 20 years, $P'(20) = .2 \cdot e^{.004(20)} = .2 \cdot e^{.08} = .2166$ thousand people/yr

$= \boxed{217 \text{ people/year}}$
(# added in 21st year)

Note units —
it is a <u>rate</u>.