**Mathematics 108**
**Big Quiz (Sections A.1, A.2 & 1.1-1.3)**

Name: ANSWER KEY A
February 6, 2009

**Use of Calculators is NOT permitted on this quiz.** Please put all answers on this page, and in boxes or on the lines provided. Show all work **neatly** on this sheet or on your scratch paper.

1. Rewrite each of the following expressions, changing to exponential form if necessary, and using a single exponent:

   a) $(a^4)^6 = a^{4\cdot 6} = \boxed{a^{24}}$

   b) $(\sqrt{x})^5 = (x^{1/2})^5 = \boxed{x^{5/2}}$

   c) $\dfrac{y^{-8}}{y^{-3}} = y^{-8-(-3)} = y^{-8+3} = \boxed{y^{-5} \text{ or } \dfrac{1}{y^5}}$

   d) $(x+7)(x+7)(x+7)(x+7) = (x+7)^4$

2. Solve for $x$: $x^5 = 9x^3$

   $x^5 - 9x^3 = 0$

   $x^3(x^2-9) = 0$

   $x^3(x+3)(x-3) = 0$

   $\boxed{x=0 \quad x=-3 \quad x=3}$

3. Let $f(x) = 2x^2 + x - 1$. Calculate and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$

$$\frac{2(x+h)^2+(x+h)-1-(2x^2+x-1)}{h} = \frac{2(x^2+2xh+h^2)+x+h-1-2x^2-x+1}{h} = \frac{2x^2+4xh+2h^2+h-2x^2}{h}$$

$$= \frac{h(4x+2h+1)}{h} = \boxed{4x+2h+1}$$

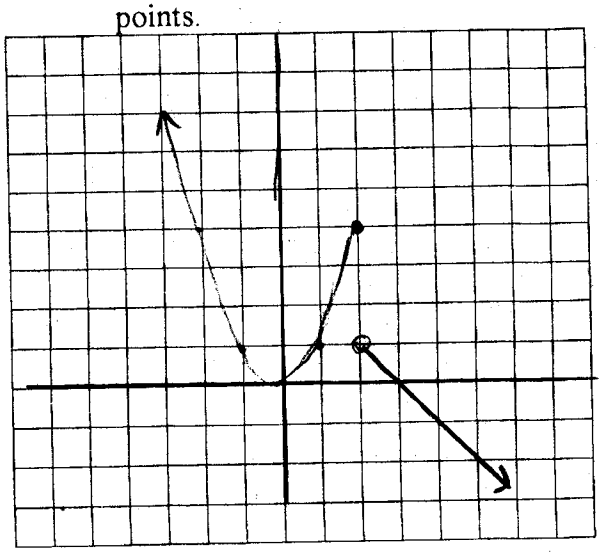
4. Let $h(x) = \begin{cases} x^2, & \text{if } x \le 2 \\ -x+3, & \text{if } x > 2 \end{cases}$

   a) Calculate: $h(-1) = (-1)^2 = 1$

   $h(2) = 2^2 = 4$

   $h(3) = -3+3 = 0$

   b) Graph the function $h(x)$ below. Be sure to label your axes clearly and identify all important points.



(Note: find endpoints of both intervals)

5. Given the functions $f(x)=\sqrt{x+3}$ and $g(x)=\dfrac{x^2}{x-4}$, find the following. You do not need to simplify your answers. Extra credit for correctly finding the domain of the composed function.

a) $f(g(x))=f\left(\frac{x^2}{x-4}\right)=\sqrt{\frac{x^2}{x-4}+3}=\sqrt{\frac{x^2+3(x-4)}{x-4}}=\sqrt{\frac{x^2+3x-12}{x-4}}$ Domain $\{x \mid x \le \frac{-3-\sqrt{57}}{2}$ or $\frac{-3+\sqrt{57}}{2} \le x < 4$ or $x>4\}$

b) $g(f(x))=g\left(\sqrt{x+3}\right)=\dfrac{(\sqrt{x+3})^2}{(\sqrt{x+3}-4}$ Domain: $\{x \ge -3, x \ne 13\}$

$x^2+3x-12=0$, $x=\frac{-3\pm\sqrt{57}}{2}$; $x-4=0$, $x=4$

| | $-6$ | $\frac{-3-\sqrt{57}}{2}$ | | $\frac{-3+\sqrt{57}}{2}$ | | $4$ | |
|---|---|---|---|---|---|---|---|
| $x^2+3x-12$ | + | 0 | − | 0 | + | | + |
| $x-4$ | − | | − | | − | 0 | + |
| | − | 0 | + | 0 | − | | + |

6. What is the **domain** of each of the following functions?

a) $f(x)=\sqrt{2x+6}$ Domain = $\{x \mid x \ge -3\}$

$2x+6 \ge 0$ $2x \ge -6$

b) $g(x)=\dfrac{-4x}{x^2+1}$ Domain = $\mathbb{R}$ $\{x^2+1$ Never equals $0\}$

c) $f(x)=\dfrac{2x+1}{x^2+5x-2}$ Domain = $\left\{x \mid x \ne \frac{-5\pm\sqrt{33}}{2}\right\}$

let $x^2+5x-2=0$, $x=\frac{-5\pm\sqrt{25-4(1)(-2)}}{2}=\frac{-5\pm\sqrt{33}}{2}$

7. Consider the points $(-1,2)$ and $(3,-10)$:

a) What is the slope of the line through the two points?

$\frac{2-1-10}{-1-3}=\frac{-4}{-4}=-3=m$

b) What is the equation, in slope-intercept form, of the line through the points?

$2=-3(-1)+b=3+b$; $b=-1$ $y=-3x-1$ or $y-2=-3(x+1)=-3x-3$, $y=-3x-1$

c) What is an equation (in any form) of the line through (2, 4) and parallel to the line above?

$y-4=-3(x-2)$ or $y-4=-3x+6$; $y=-3x+10$

8. Consider the function $f(x)=x^2-6x+8$. Find the following:

2 a) $x$-intercept(s) $x=4$, $x=2$ $(x-4)(x-2)=0$ Let $x-4=0$ $x=4$; $x-2=0$ $x=2$

1 b) $y$-intercept $(0,8)$ $f(0)=0^2-6(0)+8=8$

2 c) vertex (both coordinates) $(3,-1)$ $x=\frac{-b}{2a}=\frac{-(-6)}{2}=3$. $f(3)=9-18+8=-1$

d) graph the function $f(x)=x^2-6x+8$.

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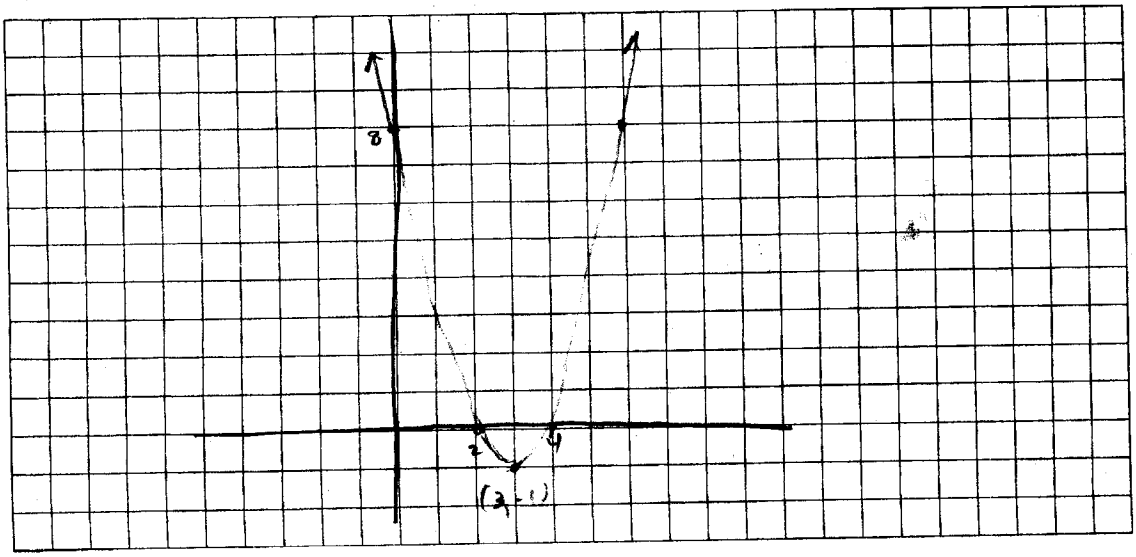


**Mathematics 108**
**Big Quiz (Sections A.1, A.2 & 1.1-1.3)**

Name: ANSWER KEY B
February 6, 2009

**Use of Calculators is NOT permitted on this quiz.** Please put all answers on this page, and in boxes or on the lines provided. Show all work **neatly** on this sheet or on your scratch paper.

2 pts. ea.

1. Rewrite each of the following expressions, changing to exponential form if necessary, and using a single exponent:

   a) $\dfrac{x^{-7}}{x^{-3}} = x^{-7+3} = \boxed{x^{-4} \text{ or } \dfrac{1}{x^4}}$

   b) $\left(b^3\right)^5 = b^{3\cdot 5} = \boxed{b^{15}}$

   c) $(x-3)(x-3)(x-3)(x-3)(x-3) = \boxed{(x-3)^5}$

   d) $\left(\sqrt{y}\right)^7 = \left(y^{1/2}\right)^7 = \boxed{y^{7/2}}$

4 pts

2. Solve for $x$: $x^4 = 16x^2$

   $x^4 - 16x^2 = 0$

   $x^2(x^2-16) = 0$

   $x^2(x+4)(x-4) = 0$

   $\boxed{x = 0 \quad x = -4 \quad x = 4}$ (2, 1, 1)

4 pts

3. Let $f(x) = 3x^2 + x - 2$. Calculate and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.

   $\dfrac{3(x+h)^2 + (x+h) - 2 - (3x^2 + x - 2)}{h} = \dfrac{3(x^2 + 2xh + h^2) + x + h - 2 - 3x^2 - x + 2}{h}$

   $= \dfrac{3x^2 + 6xh + 3h^2 + h - 3x^2}{h}$

   $= \dfrac{h(6x + 3h + 1)}{h}$

   $= \boxed{6x + 3h + 1}$

4. Let $g(x) = \begin{cases} x^2, & \text{if } x \le 1 \\ -x + 4, & \text{if } x > 1 \end{cases}$

   a) Calculate: (2)

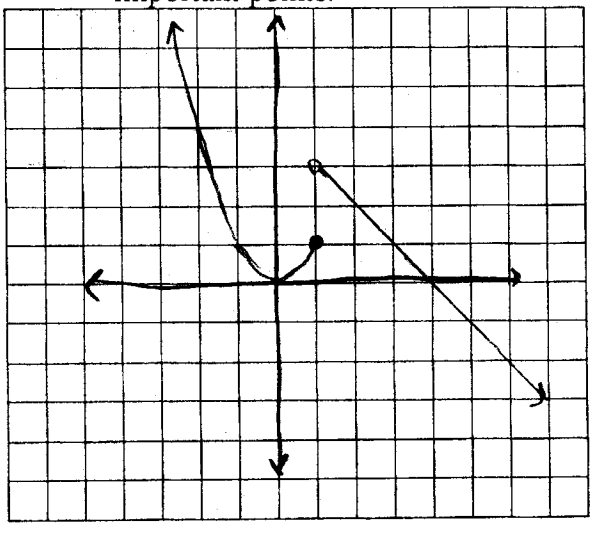
   $g(2) = -2 + 4 = 2$

   $g(1) = 1^2 = 1$ (2)

   $g(-2) = (-2)^2 = 4$ (2)

   b) Graph the function $g(x)$ below. Be sure to label your axes clearly and identify all important points.

4 pts.



26

2 pts. ea.

5. Given the functions $f(x)=\dfrac{x^2}{1-x}$ and $g(x)=\sqrt{x-2}$, find the following. You do not need to simplify your answers. Extra credit for correctly finding the domain of the composed function.

a) $f(g(x)) = f(\sqrt{x-2}) = \left|\dfrac{(\sqrt{x-2})^2}{1-\sqrt{x-2}}\right| = \dfrac{x-2}{1-\sqrt{x-2}}$ (Domain: $\{x \mid x \geq 2, x \neq 3\}$)

b) $g(f(x)) = g\left(\dfrac{x^2}{1-x}\right) = \sqrt{\dfrac{x^2}{1-x} - 2}$ (Domain: $\{x \mid x \leq -1-\sqrt{3}$ or $-1+\sqrt{3} \leq x < 1\}$)

$x^2+2x-2+$ | $-0$ | $3/4$ | $2$
$1-x$ | $+$ | $+$ | $+$ | $-$
$-1-\sqrt{3}$ $-1+\sqrt{3}$ 1
$+$ $-$ $+$ $-$

6 pts.

6. What is the **domain** of each of the following functions?

a) $f(x)=\dfrac{2x+1}{x^2+7x-1}$ Domain = $\{x \mid x \neq -7 \pm \sqrt{53}\}$

Let $x^2+7x-1=0$

$x = \dfrac{-7 \pm \sqrt{49-4(1)(-1)}}{2}$

$= \dfrac{-7 \pm \sqrt{53}}{2}$

b) $g(x)=\dfrac{-x}{x^2+4}$ Domain = $\mathbb{R}$ ($x^2+4$ to for any real value of $x$)

c) $f(x)=\sqrt{3x+6}$ Domain = $\{x \mid x \geq -2\}$

Let $3x+6 \geq 0$

$3x \geq -6$

$x \geq -2$

6 pts.

7. Consider the points $(-1,1)$ and $(2,-11)$:

a) What is the slope of the line through the two points?

$\dfrac{-11-1}{2-(-1)} = \dfrac{-12}{3} = -4 = m$

b) What is the equation, in slope-intercept form, of the line through the points?

$y-1 = -4(x+1) = -4x-4 \Rightarrow y = -4x-3$ or solve for $b$ using either point.

c) What is an equation (in any form) of the line through (3, 4) and parallel to the line above?

$y-4 = -4(x-3)$, or $y = -4x+16$

8. Consider the function $f(x)=x^2+6x+8$. Find the following:

a) $x$-intercept(s) $x=-4, x=-2$ $x^2+6x+8=0$ $(x+4)(x+2)=0$

$x=-4$ $x=-2$

5

b) $y$-intercept $y=8$ (or $(0,8)$) $f(0)=0^2+6\cdot 0+8$

c) vertex (both coordinates) $(-3,-1)$ $-\dfrac{b}{2a} = \dfrac{-6}{2\cdot 1} = -3 = x$ $f(-3)=(-3)^2+6(-3)+8$

$= 9-18+8 = -1 = y$

d) graph the function:

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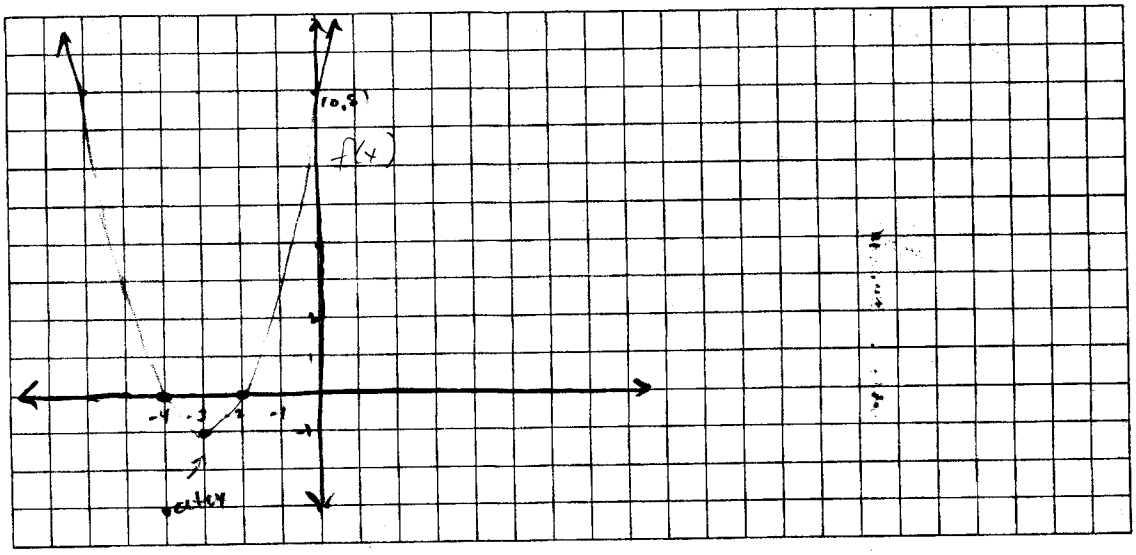


24