15 pts - 3 each.

Mathematics 108
Quiz, Sections 2.3

Name ANSWER KEY A
March 27, 2009

No calculators allowed. Show all work neatly. You do not need to simplify your answers, but you may do any correct algebraic simplifications before you find the indicated derivatives.

1. Find the first derivative of each of the following functions, using any correct technique.

a) $f(x) = (x^5 + 2x)(3x^4 - 4x^2 + 7)$

$f'(x) = (5x^4+2)(3x^4-4x^2+7) + (12x^3-8x)(x^5+2x)$

OR

$f'(x) = 28x^8 - 28x^6 + 65x^4 - 24x^2 + 14$

2 possible approaches:

I. Product Rule

$f = x^5+2x \quad g = 3x^4-4x^2+7$

$f' = 5x^4+2 \quad g' = 12x^3-8x$

$f'(x) = (5x^4+2)(3x^4-4x^2+7) + (12x^3-8x)(x^5+2x)$

OR II. Expand:

$f(x) = 3x^9 - 4x^7 + 7x^5 + 6x^5 - 8x^3 + 14x$

$= 3x^9 - 4x^7 + 13x^5 - 8x^3 + 14x$

$f'(x) = 27x^8 - 28x^6 + 65x^4 - 24x^2 + 14$

b) $f(x) = \dfrac{6x^3 + 7x}{x^2 - 2}$

$f'(x) = \dfrac{(18x^2+7)(x^2-2) - 2x(6x^3+7x)}{(x^2-2)^2}$

OR

$\dfrac{18x^4 - 36x^2 + 7x^2 - 14 - 12x^4 - 14x^2}{(x-2)^2}$

$= \dfrac{6x^4 - 43x^2 - 14}{(x-2)^2}$

Must use Quotient rule.

$f = 6x^3+7x \quad g = x^2-2$

$f' = 18x^2+7 \quad g' = 2x$

c) $f(x) = (5x-2)(\sqrt{x})$

$f'(x) = \dfrac{\frac{15}{2}x^{1/2} - x^{-1/2}}{}$

OR

$f'(x) = 5x^{1/2} + (5x-2)(\frac{1}{2}x^{-1/2})$

2 possible approaches.

I. Expand:

$f(x) = (5x-2)(x^{1/2}) = 5x^{3/2} - 2x^{1/2}$

$f'(x) = 5 \cdot \frac{3}{2} \cdot x^{1/2} - 2 \cdot \frac{1}{2} \cdot x^{-1/2} = \frac{15}{2}x^{1/2} - x^{-1/2}$

or $7\frac{1}{2}x^{1/2} - x^{-1/2}$

OR II. Product rule

$f = 5x-2 \quad g = x^{1/2}$

$f' = 5 \quad g' = \frac{1}{2}x^{-1/2}$

2. Find the first and second derivatives of the function: $f(x) = x^3(7x^2 - 3x)$

$f'(x) = 35x^4 - 12x^3 = 3x^2(7x^2-3x) + x^3(14x-3)$

$f''(x) = 140x^3 - 36x^2 = $ (see below)

Multiple approaches:

a) Expand: $f(x) = 7x^5 - 3x^4$

$f'(x) = 35x^4 - 12x^3$

$f''(x) = 140x^3 - 36x^2$

OR: Product rule:

$f = x^3 \quad g = 7x^2 - 3x \quad f'(x) = 3x^2(7x^2-3x) + x^3(14x-3)$ ← leave like this or simplify to answer above.

$f' = 3x^2 \quad g' = 14x-3$

$f''(x)$ - either end up with answer above or use product rule 2x.

$f''(x) = 6x(7x^2-3x) + (14x-3)(3x^2) + 3x^2(14x-3) + x^3(14)$ ←

Mathematics 108
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Name ANSWER KEY B
March 27, 2009

No calculators allowed. Show all work neatly. You do not need to simplify your answers, but you may do any correct algebraic simplifications before you find the indicated derivatives.

1. Find the first derivative of each of the following functions, using any correct technique.

a) $f(x) = (2x^3 - 7x^2 + 12)(x^4 + 3x)$

2 approaches:

I. Product Rule

$f = 2x^3 - 7x^2 + 12 \quad g = x^4 + 3x$

$f' = 6x^2 - 14x \quad g' = 4x^3 + 3$

$f'(x) = (6x^2 - 14x)(x^4 + 3x) + (4x^3 + 3)(2x^3 - 7x^2 + 12)$

$f'(x) = \underline{(6x^2 - 14x)(x^4 + 3x) + (2x^3 - 7x^2 + 12)(4x^3 + 3)}$

OR $f'(x) = 14x^6 - 42x^5 + 72x^3 - 63x^2 + 36$

OR: Expand:

$f(x) = 2x^7 + 6x^4 - 7x^6 - 21x^3 + 12x^4 + 36x$

$= 2x^7 - 7x^6 + 18x^4 - 21x^3 + 36x$

$f'(x) = 14x^6 - 42x^5 + 72x^3 - 63x^2 + 36$

b) $f(x) = \dfrac{2x^5 + 3x}{x^2 - 4}$

Must use Quotient Rule

$f = 2x^5 + 3x \quad g = x^2 - 4$

$f' = 10x^4 + 3 \quad g' = 2x$

$f'(x) = \dfrac{(10x^4 + 3)(x^2 - 4) - 2x(2x^5 + 3x)}{(x^2 - 4)^2}$

OR $\dfrac{(10x^6 - 40x^4 + 3x^2 - 12 - 4x^6 - 6x^2)}{(x^2 - 4)^2}$

$\dfrac{6x^6 - 40x^4 - 3x^2 - 12}{(x^2 - 4)^2}$

c) $f(x) = (\sqrt{x})(2x - 4)$

I. Expand

$f(x) = x^{1/2}(2x - 4) = 2x^{3/2} - 4x^{1/2}$

$f'(x) = 2 \cdot \frac{3}{2} \cdot x^{1/2} - 4 \cdot \frac{1}{2} \cdot x^{-1/2}$

$= 3x^{1/2} - 2x^{-1/2}$

II. Product Rule

OR $f = x^{1/2} \quad g = 2x - 4$

$f' = \frac{1}{2}x^{-1/2} \quad g' = 2$

$f'(x) = 3x^{1/2} - 2x^{-1/2}$

OR $f'(x) = \frac{1}{2}x^{-1/2}(2x - 4) + 2x^{1/2}$

2. Find the first and second derivatives of the function: $f(x) = x^2(10x^3 - 5x)$

I. Expand

$f(x) = 10x^5 - 5x^3$

$f'(x) = 50x^4 - 15x^2$

$f''(x) = 200x^3 - 30x$

$f'(x) = \underline{50x^4 - 15x^2} = 2x(10x^3 - 5x) + x^2(30x^2 - 5)$

$f''(x) = \underline{200x^3 - 30x} = $ (see below)

OR: Product Rule

$f = x^2 \quad g = 10x^3 - 5x$

$f' = 2x \quad g' = 30x^2 - 5$

$f'(x) = 2x(10x^3 - 5x) + x^2(30x^2 - 5)$. To find 2nd derivative, simplify to $f'(x)$ above or use product rule 2 times.

$f''(x) = 2(10x^3 - 5x) + 2x(30x^2 - 5) + 2x(30x^2 - 5) + x^2(60x)$