Mathematics 108 Name: ANSWER KEY A

Chapter 2 Test April 3, 2009

No calculators allowed. Show all work neatly. Please put your answers in the spaces provided or in $\boxed{\text{boxes}}$

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1. Use the limit definition of the derivative, that is $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, to find the derivative of $f(x) = 5x - 3x^2$. You may check your answer with the power rule, but you must show all steps of your reasoning to get any credit for this answer.

$f'(x) = \lim_{h\to 0} \frac{5(x+h) - 3\overbrace{(x+h)^2}^{3(x^2+2xh+h^2)} - (5x-3x^2)}{h} = \lim_{h\to 0} \frac{\cancel{5x} + 5h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{5x} + \cancel{3x^2}}{h}$

$= \lim_{h\to 0} \frac{5h - 6xh - 3h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(5-6x-3h)}{\cancel{h}} = \lim_{h\to 0} 5 - 6x - \cancel{3h}^{\to 0} = \boxed{5-6x}$

7.8=56 8 ea.

2. Find the following derivatives, using any correct techniques. Unless otherwise specified, you do not have to simplify your answers.

a) $f(x) = \frac{4x-3}{x^2+4}$ Use Quotient Rule. $f'(x) = \frac{-4x^2+6x+16}{(x^2+4)^2}$

(simplify the numerator)

$f'(x) = \frac{4(x^2+4) - 2x(4x-3)}{(x^2+4)^2}$ $f = 4x-3$ $g = x^2+4$

$f' = 4$ $g' = 2x$

$= \frac{4x^2+16-8x^2+6x}{(x^2+4)^2}$

b) $f(x) = (x^2-2x)(5-\frac{1}{x^2})$ expand $= 5x^2 - 1 - 10x + 2x^{-1}$ $f'(x) =$ expand first $10x - 10 - 2x^{-2}$ OR

$f'(x) = (2x-2)(5-x^{-2}) + 2x^{-3}(x^2-2x)$ product rule

product rule: $f = x^2-2x$ $g = 5 - x^{-2}$

$f' = 2x-2$ $g' = 2x^{-3}$

c) $f(x) = \sqrt[4]{3x^2-4} = (3x^2-4)^{1/4}$ $f'(x) = \frac{3}{2}x(3x^2-4)^{-3/4}$

$f'(x) = \frac{1}{4}(3x^2-4)^{-3/4}(6x) = \frac{6x}{4}(3x^2-4)^{-3/4}$

chain rule

3. Find the first and second derivatives of each of the following:

a) $f(x) = 2x^2 + \frac{7}{x} = 2x^2 + 7x^{-1}$ $f'(x) = 4x - 7x^{-2}$

$f''(x) = 4 + 14x^{-3}$

b) $f(x) = (x^4-2)^6$ $f'(x) = 6(x^4-2)^5(4x^3)$ (chain rule) $= 24x^3(x^4-2)^5$

$f''(x) = 24x^3(20x^3)(x^4-2)^4 + 72x^2(x^4-2)^5$

$= 480x^6(x^4-2)^4 + 72x^2(x^4-2)^5$

$f = 24x^3$ $g = (x^4-2)^5$

$f' = 72x^2$ $g' = 5(x^4-2)^4(4x^3)$

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$= 2x^3 - 3x^6$, so $f'(x) = 6x^2 - 18x^5$

14 pts.

4. Find an equation of the line tangent to $f(x) = x^3(2-3x^3)$ at $x=1$.

3 I. Point: plug $x=1$ into $f(x)$ to get y: $f(1) = 1^3(2-3\cdot1^3) = -1$ | $\underline{(1,-1)}$ is the point.

4 II. Slope: find $f'(x) = 3x^2(2-3x^3) + (-9x^2)(x^3)$ (product Rule or $f = x^3$, $g = 2-3x^3$, $f' = 3x^2$, $g' = -9x^2$

4 $f'(1) = m_{tan} = 3\cdot1(2-3(1)^3) - 9(1) = 3(-1) - 9 = -3 - 9 = -12 = m_{tan}$

3 III. Line: $y-(-1) = -12(x-1)$; $y+1 = -12x+12$; $y = -12x+11$

12 pts.

5. An upscale fast food restaurant has determined that the relationship between the price p, in dollars, at which it can sell hamburgers and the quantity q that it can sell is $p = \dfrac{80{,}000-q}{20{,}000}$.

a) Find the revenue function as a function of the quantity of hamburgers sold (ie, find $R(q)$).

$R(q) = q\cdot\left(\frac{80{,}000-q}{20{,}000}\right) = \frac{80{,}000q - q^2}{20{,}000} = \frac{1}{20{,}000}(80{,}000q - q^2)$

b) Find the marginal revenue function.

$R'(q) = \frac{1}{20{,}000}\cdot(80{,}000-2q) = \frac{80000-2q}{20000}$

c) Use the marginal revenue function to estimate the revenue from the sale of the 10,001st hamburger and interpret your result (tell me in words, with units, what your answer means.)

$R'(10{,}000) = \frac{1}{20{,}000}(80{,}000 - 2(10{,}000)) = \frac{1}{20{,}000}(80{,}000-20{,}000) = \frac{60{,}000}{20000} = \$3/\text{hamburger}$

10 pts.

6. When a company produces and sells x thousand units per week, its total weekly profit is P thousand dollars, where $P = \dfrac{300x}{200+x^2}$.

The production level, in thousands of units, at t weeks from the present is $x = 2+3t$.

Find the function that models how fast profits are changing with respect to time (that is, find $\dfrac{dP}{dt}$). You do not need to simplify your answer.

Approach 1: expand first.

$P(t) = \frac{300(2+3t)}{200+(2+3t)^2} = \frac{600+900t}{200+4+12t+9t^2} = \frac{600+900t}{204+12t+9t^2}$

36 $P'(t) = \frac{900(204+12t+9t^2) - (12+18t)(600+900t)}{(204+12t+9t^2)^2}$

Approach 2: Quotient Rule directly

$\frac{300(2+3t)}{200+(2+3t)^2} = P(t)$

$P'(t) = \frac{300\cdot3\cdot[200+(2+3t)^2] - (300)(2+3t)(2(2+3t)(3))}{[200+(2+3t)^2]^2}$

(Extra credit: What happens to profits in the long run? How do you know?)

(this is asking $\lim_{t\to\infty} \frac{600+900t}{204+12t+9t^2} = 0$ (profits approach \$0 as $t\to\infty$ (power in denominator is > power in numerator)