Math 108 - Homework #5 ANSWER KEY

Section 4.1

18. a) $(x^2 y^{-3} z)^3 = x^6 y^{-9} z^3 = \dfrac{x^6 z^3}{y^9}$

(Simplification generally implies "get rid of negative exponents")

b) $\left(\dfrac{x^3 y^{-2}}{z^4}\right)^{1/6} = \dfrac{x^{1/2} y^{-1/3}}{z^{2/3}} = \dfrac{x^{1/2}}{y^{1/3} z^{2/3}}$

22. $4^x (1/2)^{3x} = 8$

Many ways to solve: one way is express everything as powers of 2: ($4 = 2^2$; $1/2 = 2^{-1}$, $8 = 2^3$)

so: $(2^2)^x (2^{-1})^{3x} = 2^3$

$2^{2x} \cdot 2^{-3x} = 2^3$

$2^{-x} = 2^3$, so $-x = 3$

$\boxed{x = -3}$

28. $B(t) = (1+\frac{r}{n})^{nt}$, if interest is compounded periodically; $B(t) = Pe^{rt}$ if comp. continuously

a. annually ($n=1$) so $B(10) = 5000(1+\frac{.10}{1})^{1 \cdot 10} = 5000(1.10)^{10} = \$12,968.712$

$\approx \boxed{\$12,968.71}$

b. semiannually ($n=2$) so $B(10) = 5000(1+\frac{.10}{2})^{2 \cdot 10} = 5000(1.05)^{20} = 13,266.488$

$\boxed{\approx \$13,266.49}$

c. Daily ($n=365$), so $B(10) = 5000(1+\frac{.10}{365})^{365 \cdot 10} = \$13589.547 \approx$

$\boxed{\$13,589.55}$

d. continuously: $B(10) = 5000(e^{.10(10)}) = 5000 \cdot e = \$13,591.409$

$\approx \boxed{\$13,591.41}$

40. a) $GDP = \$500(1+.027)^t$ billion

b) $GDP(10) = \$500(1.027)^{10} = 652.6 \sim \$653$ billion.

Section 4.3

6. $f(x) = e^{x^2+2x-1}$

$f'(x) = e^{x^2+2x-1}(2x+2)$

10. $f(x) = \sqrt{1+e^x} = (1+e^x)^{1/2}$

$f'(x) = \frac{1}{2}(1+e^x)^{-1/2}(e^x) = \frac{e^x}{2}(1+e^x)^{-1/2}$

28. $h(x) = \dfrac{e^{-x}}{x^2}$ (Quotient)

$f = e^{-x}$ $\quad g = x^2$

$f' = e^{-x}(-1) = -e^{-x}$ $\quad g' = 2x$

$$h'(x) = \frac{-e^{-x}(x^2) - 2x(e^{x})}{x^4} = \frac{-e^{-x}(x)[x+2]}{x^4} = \frac{-e^{-x}(x+2)}{x^3}$$

36. $F(x) = e^{x^2-2x}$

$F'(x) = e^{x^2-2x}(2x-2)$

Let $F'(x) = 0 = \underbrace{e^{x^2-2x}(2)}_{\text{Never}=0}\underbrace{(x-1)}$ $\quad x = 1$ is only CV.

on interval $[0,2]$, find $\left.\begin{array}{l} F(0) = e^0 = 1 \\ F(2) = e^{4-4} = e^0 = 1 \end{array}\right\}$ abs max

$F(1) = e^{1-2} = e^{-1} = \frac{1}{e}$ ← abs min

(betw $1/2$ & $1/3$)

max at $(0,1)$ and $(2,1)$

min at $(1, 1/e)$

72. a) $C(t) = 0.12t \cdot e^{-t/2}$ (a product)

$f = 0.12t$ $\quad g = e^{-t/2}$

$f' = 0.12$ $\quad g' = e^{-t/2}(-1/2)$

$C'(t) = 0.12(e^{-t/2}) + 0.12t(e^{-t/2})(-1/2)$

$= 0.12e^{-t/2}(1 - t/2)$

b) Find CV: let $1 - t/2 = 0$

$t/2 = 1$

$t = 2$

	$t<2$	2	$t>2$
$0.12e^{-t/2}$	+		+
$1 - t/2$	+		−
	+ ↗		− ↘

Begins to decrease after 2 hours.