Mathematics 108
Quiz, Section 3.1

Name ANSWER KEY
April 17, 2009

Due at the beginning of class on Monday, April 20, 2009

- You may use notes, your book and a calculator to help you answer the questions on this quiz, but you must show all steps of your work (a graph or answer copied from a calculator will not get you any credit.)
- You *may not* communicate with any other person about this quiz.
- Any evidence that your answers are not completely your own will be dealt with as harshly as possible.
- Your answers must be neat and legible. I will only grade answers that I can read easily.

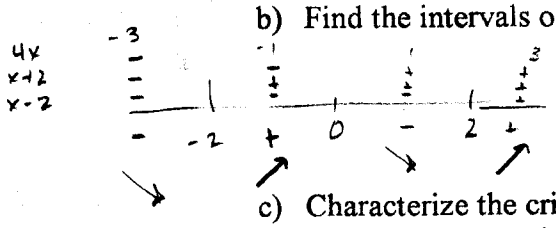
1. Consider the function: $f(x) = x^4 - 8x^2 + 4$.
   a) Find all critical values of $f(x)$. 1. find $f'(x) = 4x^3 - 16x$

2. let $f'(x) = 0 = 4x^3 - 16x = 4x(x^2-4) = 4x(x+2)(x-2)$

$4x = 0 \quad x+2 = 0 \quad x-2 = 0$

$x = 0 \quad x = -2 \quad x = 2$ ← critical values

   b) Find the intervals of increase and decrease for $f(x)$.



intervals of increase: $(-2, 0) \cup (2, \infty)$
intervals of decrease: $(-\infty, -2) \cup (0, 2)$

   c) Characterize the critical values as relative maxima, relative minima, or neither. Show your reasoning clearly.

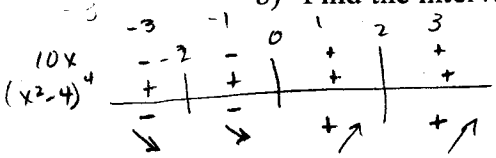
$x = -2$ is a relative minimum
$x = 0$ is a relative maximum
$x = 2$ is a relative minimum
} from 1st derivative test (direction of the function changes at each CV)

2. Consider the function: $f(x) = (x^2 - 4)^5$.
   a) Find all critical values of $f(x)$. 1. find $f'(x) = 5(x^2-4)^4(2x)$

let $5(x^2-4)^4(2x) = 0 = 5[(x+2)(x-2)]^4 \cdot 2x = 0$

CV's at $x = -2, \ x = 2, \ x = 0$

   b) Find the intervals of increase and decrease for $f(x)$.



intervals of decrease: $(-\infty, -2), (-2, 0)$
intervals of increase: $(0, 2), (2, \infty)$

   c) Characterize the critical values as relative maxima, relative minima, or neither. Show your reasoning clearly.

$x = -2$ and $x = 2$ are neither maxima nor minima (function doesn't change direction there)

$x = 0$ is a relative minimum (function goes from decreasing to increasing)

3. Consider the function $f(t) = \dfrac{t^2}{2-t}$.

a) Find the points of discontinuity, if any, of $f(t)$.

Let $2-t=0$; $t=0$ is the only point of discontinuity.

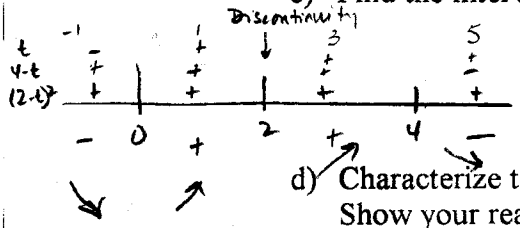
b) Find the critical values, if any, of $f(t)$.

Find $f'(t) = \dfrac{2t(2-t)-(-1)(t^2)}{(2-t)^2}$ $\quad f=t^2 \quad g=2-t$ $\quad f'=2t \quad g'=-1$

$= \dfrac{4t-2t^2+t^2}{(2-t)^2} = \dfrac{4t-t^2}{(2-t)^2} = \dfrac{t(4-t)}{(2-t)^2}$ CV's occur when $t(4-t)=0$

$t=0 \quad t=4$

c) Find the intervals of increase and decrease for $f(t)$.



decreasing: $(-\infty, 0)$ and $(4, +\infty)$

increasing $(0,2)$ and $(2,4)$

d) Characterize the critical values as relative maxima, relative minima, or neither. Show your reasoning clearly.

Rel. minimum at $x=0$

Rel. maximum at $x=4$

} from 1st derivative test / changes in direction above.

discontinuity at $x=2$ (vertical asymptote)

4. Do problem 62, page 202 (all parts a-c).

$P(r) = \dfrac{5(3r+1)}{r^2+r+2} = \dfrac{15r+5}{r^2+r+2}$

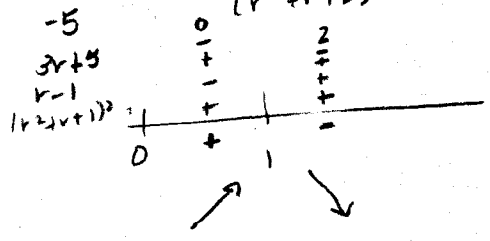
a) at city center, $r=0$, so $P(r) = \dfrac{5(3\cdot 0+1)}{0+0+2} = \dfrac{5}{2} = 2.5$ hundred $= 250$ people.

b) Find $P'(r) = \dfrac{15(r^2+r+2) - (2r+1)(15r+5)}{(r^2+r+2)^2} = \dfrac{15r^2+15r+30-30r^2-25r-5}{(r^2+r+2)^2}$ $\quad (30r^2+25r+5)$

$= \dfrac{-15r^2-10r+25}{(r^2+r+2)^2} = \dfrac{-5(3r^2+2r-5)}{(r^2+r+2)^2} = \dfrac{-5(3r+5)(r-1)}{(r^2+r+1)^2}$ Let $(3r+5)(r-1)=0$

$r=-5/3 \quad r=1$

Not in domain



increasing on $[0,1)$
decreasing on $(1, +\infty)$

c. Population is greatest at r=1mile

at $r=1$, $P(1) = \dfrac{15+5}{1+1+2} = \dfrac{20}{4} = 500$ people.