Mathematics 108, Introductory Calculus
Test 1, Sections 1.4-1.6

Name ANSWER KEY A
February 27, 2009

Show all work neatly. Use of calculators is **not** permitted on this test.

24 pts (6 ea)

1. Consider the function: $f(x) = \frac{10x + 2x^2}{x^2 - 25}$. Find each of the following limits, if the limit exists. If the limit does not exist, write DNE.

a) $\lim_{x\to 0} f(x) = \boxed{0}$ ✓

plug in: $\frac{10\cdot 0 + 2\cdot 0}{0 - 25} = \frac{0}{-25} = 0$

b) $\lim_{x\to 5} f(x) = \boxed{DNE}$

Plug in: $\frac{50+50}{25-25} = \frac{100}{0}$

c) $\lim_{x\to -5} f(x) = \lim_{x\to -5} \frac{2x\cancel{(5+x)}}{(x-5)\cancel{(x+5)}} = \frac{-10}{-10} = \boxed{1}$

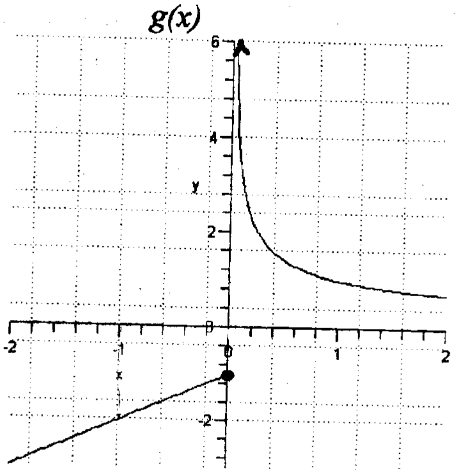
$\frac{-50+50}{25-25} = \frac{0}{0}$; keep going

d) $\lim_{x\to \infty} f(x) = \lim_{x\to\infty} \frac{2x^2+10x}{x^2-25} = \boxed{2}$

$= \lim_{x\to\infty} \frac{\frac{10x}{x^2} + \frac{2x^2}{x^2}}{\frac{x^2}{x^2} - \frac{25}{x^2}} = \lim_{x\to\infty} \frac{\overset{\nearrow 0}{\frac{10}{x}} + 2}{1 - \underset{\downarrow 0}{\frac{25}{x^2}}} = \frac{2}{1} = \boxed{2}$

15 pts. (2½ ea)

2. Given the graph of $g(x)$, find the following (if they exist).



a) $\lim_{x\to -1} g(x) = -2$

b) Is $g(x)$ continuous at $x = -1$? yes
Briefly explain why or why not. No problems there: no jumps or holes ($\lim_{x\to -1} g(x) = f(-1) = -2$)

c) $\lim_{x\to 0^+} g(x) =$ DNE (vertical asymptote)

d) $\lim_{x\to 0^-} g(x) = -1$

e) $\lim_{x\to 0} g(x) =$ DNE

f) $g(0) = -1$

g) Is $g(x)$ continuous at $x = 0$? NO Briefly explain why or why not.
The limit does not exist ($\lim_{x\to 0} g(x)$ DNE)

✓ 39
42
/81

6 pts.

3. Find the value of the constant A that makes the function $f(x)$ continuous at all values of x.

$$f(x)=\begin{cases}\left(\dfrac{x^2-4x+3}{x-1}\right), & \text{if } x>1\\ Ax-4, & \text{if } x\le 1\end{cases}$$

1) find $\lim_{x\to 1}\frac{x^2-4x+3}{x-1} = \lim_{x\to 1}\frac{(\cancel{x-1})(x-3)}{\cancel{x-1}} = \boxed{-2}$

② set $A\cdot 1-4=-2$

$A=-2+4=\boxed{2}$

$(\lim_{x\to 1^-} 2x-4 = -2 = \lim_{x\to 1^+})$

6 pts.

4. Is the function $f(x)$ below continuous at $x=1$? No Justify your answer mathematically.

$$f(x)=\begin{cases}x-1 & \text{if } x<1\\ 1 & \text{if } x=1\\ -2x+2 & \text{if } x>1\end{cases}$$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 1-1=0$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} -2x+2=0$

$\therefore \lim_{x\to 1} f(x)=0$, but $f(1)=1 \Rightarrow$ Not continuous at $x=1$

6 pts

5. It is estimated that t years from now, the population of a certain suburban community will be $P(t)=20-\dfrac{4}{0.25(t+1)}$ thousand people.

What happens to the population in the long run, that is as $t\to+\infty$? Be sure to specify units in your answer. (long run translates to $\lim_{t\to+\infty}$)

$\lim_{x\to\infty}$ $20-\frac{4}{0.25(t+1)} = \lim_{t\to+\infty} 20-\frac{\frac{4}{t}}{0.25(\frac{t}{t}-\frac{1}{t})} = \lim_{t\to+\infty} 20-\frac{\cancel{\frac{4}{t}}\nearrow}{0.25(1-\frac{1}{t}\to 0)} = 20$; $\therefore$ Population approaches, (but does not reach) 20,000 people.

24

6. The market research department of Super Skateboards, Inc, has determined that the company sells about 200 of its MongoBoards per month, and the typical selling price of each board is \$40. The department believes that for every decrease of \$2 in the price of a board, the number sold will increase by 40 boards per month.

a) Find the linear demand (quantity) function that models the facts above. Express the demand for the MongoBoards as a function of the price p at which the board will be sold. $m=\frac{\Delta\text{ boards}}{\Delta\text{ price}}=\frac{+40}{-2}=-20$ $\quad q-200=-20(p-40)=-20p+800$

$\boxed{q=-20p+1000}$

b) Express the total revenue which Super Skateboards will receive from the sale of MongoBoards as a function of the price p of the skate boards.

$R(p)=p(-20p+1000)=-20p^2+1000p$ $\qquad -20(2500)+1000(50)$

c) Express the total profit which Super Skateboards can make from the sale of MongoBoards if the cost to produce each board is \$10.

$P(p)=\boxed{\underbrace{-20p^2+1000p}_{R(p)}-10(\underbrace{-20p+1000}_{q})}=-20p^2+1000p+200p-10,000$

$=-20p^2+1200p-10,000 \leftarrow$

d) At what price should Super Skateboards sell the MongoBoards to generate the greatest profit? $\boxed{\$30}$

Vertex: $\frac{-1200}{2(-20)}=\frac{-1200}{-40}=\$30=p$

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70 pts

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Name ANSWER KEY B
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Show all work neatly. Use of calculators is **not** permitted on this test.

24 pts

1. Consider the function: $f(x) = \frac{12x + 6x^2}{x^2 - 4}$. Find each of the following limits, if the limit exists. If the limit does not exist, write DNE.

a) $\lim_{x\to 0} f(x) = \boxed{0}$

Plug in: $\frac{12\cdot 0 + 6(0^2)}{0^2 - 4} = \frac{0}{-4} = 0$

b) $\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{6x(\cancel{2+x})}{(\cancel{x+2})(x-2)} = \frac{-12}{-4} = \boxed{3}$

plug in: $\frac{-24+24}{4-4} = \frac{0}{0}$: keep going

c) $\lim_{x\to 2} f(x) = \boxed{DNE}$

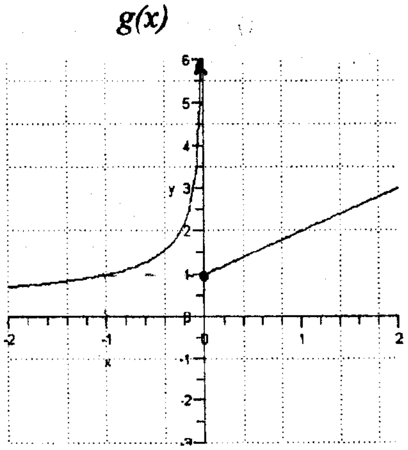
plug in: $\frac{24+24}{4-4} = \frac{48}{0}$ DNE

d) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{6x^2 + 12x}{x^2 - 4} = \boxed{6}$ ← or ↓

or $\lim_{x\to\infty} \frac{\frac{12x}{x^2} + \frac{6x^2}{x^2}}{\frac{x^2}{x^2} - \frac{4}{x^2}} = \lim_{x\to\infty} \frac{\cancel{\frac{12}{x}}\to 0 + 6}{1 - \cancel{\frac{4}{x^2}}\to 0} = \frac{6}{1} = \boxed{6}$

15 pts.

2. Given the graph of $g(x)$, find the following (if they exist).



a) $\lim_{x\to -1} g(x) =$ 1

b) Is $g(x)$ continuous at $x = -1$? yes

c) $\lim_{x\to 0^+} g(x) =$ 1

d) $\lim_{x\to 0^-} g(x) =$ DNE (vertical asymptote)

e) $\lim_{x\to 0} g(x) =$ DNE

f) $g(0) =$ 1

g) Is $g(x)$ continuous at $x = 0$? NO Briefly explain why or why not. Because $\lim_{x\to 0} g(x)$ Does Not exist.

39
28
67

3. Find the value of the constant A that makes the function $f(x)$ continuous at all values of x.

$$f(x)=\begin{cases}\left(\dfrac{x^2-3x+2}{x-1}\right), & if\ x>1\\ Ax-5, & if\ x\le 1\end{cases}$$

① find $\lim\limits_{x\to 1^+}\dfrac{x^2-3x+2}{x-1}=\lim\limits_{x\to 1^+}\dfrac{\cancel{(x-1)}(x-2)}{\cancel{(x-1)}}=-1$

6

② set $A\cdot 1-5=-1$

$A\cdot 1=4$; $\boxed{A=4}$

(Check: $\lim\limits_{x\to 1^-} 4x-5=$
$4-5=-1$

4. Is the function $f(x)$ below continuous at $x=0$? NO Justify your answer mathematically.

$$f(x)=\begin{cases}x-2 & if\ x<0\\ 1 & if\ x=0\\ 2x-2 & if\ x>0\end{cases}$$

① $\lim\limits_{x\to 0^-} f(x)=0-2=-2$

② $\lim\limits_{x\to 0^+} f(x)=0-2=-2$

6

$\therefore \lim\limits_{x\to 0} f(x)=-2$, but $f(0)=1$,

$\therefore f(x)$ is not continuous at $x=0$.

5. It is estimated that t years from now, the population of a certain suburban community will be $P(t)=30-\dfrac{4}{0.2(t+1)}$ thousand people.

6

What happens to the population in the long run, that is as $t\to+\infty$? Be sure to specify units in your answer. The population approaches but does not reach 30,000 people.

Why? Long run translates to $\lim\limits_{t\to\infty} 30-\dfrac{4}{0.2(t+1)}=\lim\limits_{t\to\infty}30+\lim\limits_{t\to\infty}\dfrac{4}{0.2t+0.2}=30+\lim\limits_{t\to\infty}\dfrac{\overset{\to 0}{\frac{4}{t}}}{\frac{0.2t}{t}+\underset{\to 0}{\frac{0.2}{t}}}=30+0=30$

10 pts.

6. The market research department of Super Skateboards, Inc, has determined that the company sells about 300 of its UberBoards per month, and the typical selling price of each board is \$90. The department believes that for every decrease of \$3 in the price of a board, the number sold will increase by 30 boards per month.

a) Find the linear demand (quantity) function that models the facts above. Express the demand for the UberBoards as a function of the price p at which the board will be sold. slope $=\dfrac{\Delta\text{ boards}}{\Delta\text{ price}}=\dfrac{30}{-3}=-10$. $q-300=-10(p-90)=-10p+900$

$\boxed{q=-10p+1200}$

b) Express the total revenue which Super Skateboards will receive from the sale of UberBoards as a function of the price p of the skate boards.

$R(p)=p\cdot q=p(-10p+1200)=-10p^2+1200p$

c) Express the total profit which Super Skateboards can make from the sale of UberBoards if the cost to produce each board is \$20.

Profit = Revenue - Cost = $-10p^2+1200p-20(-10p+1200)=-10p^2+1200p+200p-24000$

$=\boxed{-10p^2+1400p-24000}$

d) At what price should Super Skateboards sell the UberBoards to generate the greatest profit? \$70

$p=\dfrac{-1400}{2(-10)}=\dfrac{-1400}{-20}=\$70$

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