Math 108 Name ANSWER KEY

Test, Sections 2.6, 3.1, 3.2 and 3.4 April 29, 2009

Show all work neatly. You may ~~not use a~~ calculator on this exam. Please put your answers on the lines provided or in $\boxed{\text{boxes}}$

15 pts

5 1. Consider the equation $2x^2 - y^3 = 36$. Find $\frac{dy}{dx}$. You may either use implicit differentiation or solve for y and differentiate directly. $\frac{dy}{dx} = \frac{4x}{3y^2}$ or $\frac{1}{3}(2x^2-36)^{-2/3}(4x)$

$4x - 3y^2y' = 0$

$4x = 3y^2y'$

$y' = \frac{4x}{3y^2}$

OR $2x^2 - 36 = y^3$

$y = \sqrt[3]{2x^2-36} = (2x^2-36)^{1/3}$

$y' = \frac{1}{3}(2x^2-36)^{-2/3}(4x)$

10 2. Let $5x^4y - 4\sqrt{y} = 3x$ ($4y^{1/2}$). Find $\frac{dy}{dx}$, using implicit differentiation. $\frac{dy}{dx} = \frac{3-20x^3y}{5x^4-2y^{-1/2}} = \frac{20x^3y-3}{2y^{-1/2}-5x^4}$

$f = 5x^4 \quad g = y$

$f' = 20x^3 \quad g' = y'$

4 pts 3 pts

$20x^3y + 5x^4y' - 2y^{-1/2}y' = 3$

$5x^4y' - 2y^{-1/2}y' = 3 - 20x^3y$

$y' = \frac{3-20x^3y}{5x^4-2y^{-1/2}}$ 2

20 1/2

3. Consider the function $f(x) = x^5 - 5x^4 + 5x^3 + 100$.

4 a) Does the function $f(x)$ have any points of discontinuity? No If so, list them. If not, explain why. $f(x)$ is a polynomial, and polynomials are continuous everywhere.

6 b) Identify all critical values of $f(x)$. $f'(x) = 5x^4 - 20x^3 + 15x^2$

$= 5x^2(x^2-4x+3)$

$= 5x^2(x-3)(x-1)$

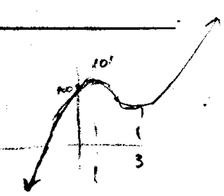
$\boxed{\text{CV's at } x=0,\ x=3,\ x=1}$

6 c) Identify the interval(s) of increase $(-\infty,0) \cup (0,1) \cup (3,+\infty)$ or $\boxed{x<0,\ 0<x<1,\ x>3}$ and the interval(s) of decrease $(1,3)$ or $\boxed{1<x<3}$

← test values

Factors	-1	1/2	2	4
$5x^2$	+	+	+	+
$(x-3)$	-	-	-	+
$(x-1)$	-	-	+	+
	+	+	-	+

0 1 3



4 1/2 d) Classify each of your critical values as a relative maximum, relative minimum, or neither.

$\boxed{\begin{array}{l} x=0: \text{ Neither} \\ x=1: \text{ relative maximum} \\ x=3: \text{ relative minimum} \end{array}}$

35 1/2

4. Consider the function $g(x) = \dfrac{x^2}{x-2}$.

4

a) Does the function $g(x)$ have any points of discontinuity? yes If so, list them. If not, explain why. Discontinuous at $x = 2$. Because the denominator would equal 0 there, the function is undefined at that point.

b) Identify all critical values of $g(x)$.

$f = x^2$ $\quad g = x - 2$

$f' = 2x$ $\quad g' = 1$

6

$g'(x) = \dfrac{2x(x-2) - 1(x^2)}{(x-2)^2} = \dfrac{2x^2 - 4x - x^2}{(x-2)^2} = \dfrac{x^2 - 4x}{(x-2)^2}$

Let $x^2 - 4x = x(x-4) = 0$; $\boxed{\text{Then } x = 0, x = 4 \text{ are CV's.}}$

6

c) Identify the interval(s) of increase $(-\infty, 0) \cup (4, +\infty)$ or $\boxed{x < 0,\ x > 4}$ and the interval(s) of decrease $(0, 2) \cup (2, 4)$ or $\boxed{0 < x < 2,\ 2 < x < 4}$.

factors	-1	CV 0	1	Disc. 2	3	CV 4	5
$x^2 - 4x$	+		−		−		+
$(x-2)^2$	+		+		+		+
	+ ↗	0	− ↘	2	− ↘	4	+ ↗

d) Classify each of your critical values as a relative maximum, relative minimum, or neither.

3

$x = 0$: relative maximum

$x = 4$: relative minimum

e) Find the absolute maximum and absolute minimum of the function $g(x)$ on the interval $3 \le x \le 6$.

6

$f(3) = \dfrac{9}{3-2} = \dfrac{9}{1} = 9$

$f(4) = \dfrac{16}{4-2} = \dfrac{16}{2} = 8$

$f(6) = \dfrac{36}{6-2} = \dfrac{36}{4} = 9$

Abs. max at $(3, 9)$ and $(6, 9)$

Abs. min at $(4, 8)$

5. An efficiency study of a day's production of sprockets at Spaceley Sprockets Company shows that the average worker who is on the job at 7:00 A.M. will have assembled $f(t) = -t^3 + 12t^2 + 27t$ sprockets t hours later.

a) At what time between 7:00 A.M. and 1:00 P.M. is the worker performing at peak efficiency?

6

$f'(t) = -3t^2 + 24t + 27$

$f''(t) = -6t + 24$

Let $f''(t) = 0 = -6t + 24$; $6t = 24$; $\boxed{t = 4 \text{ or } 11{:}00 \text{ A.M.}}$

b) At what rate is she assembling sprockets at that time? Be sure to specify your units.

4

$f'(4) = -3(4)^2 + 24(4) + 27$

$= -3(16) + 96 + 27 = -48 + 96 + 27 = 75$ sprockets/hr.

35

12

6. On a certain route, a regional airline carries 8000 passengers per month, each paying \$60. The airline wants to increase the fare, but the market research department estimates that for each \$1 increase in fare, the airline will lose 100 passengers.

a) Find the linear demand function that models the facts above, where the number of passengers per month is a function of the price p of a ticket. slope $= \frac{\Delta q}{\Delta p} = \frac{-100}{+1} = -100$

(60, 8000)

4

$q - 8000 = -100(p-60) = -100p + 6000$

$\boxed{q = -100p + 14000}$ (or $8000 = -100(60) + b = -6000 + b$, $b = 14{,}000$)

b) Find the revenue function that expresses the total monthly revenue the airline company will receive on this route as a function of the price p of a ticket.

2

$R(p) = p(-100p + 14{,}000) = -100p^2 + 14000p$

4

c) Use calculus to determine the ticket price that will maximize the airline's revenue on this route. (just need p)

$R'(p) = -200p + 14000$. Let $R'(p) = 0 = -200p + 14000$; $200p = 14000$; $p = \frac{14000}{200} = \boxed{\$70}$

2

d) How do you know that the ticket price you found maximizes revenue?

$R''(p) = -200$, so $R(p)$ is concave down everywhere, and $p = \$70$ is the only critical value, so it must be the value of p where revenue is maximized. (Also, we know it's a parabola opening down, CV must be a maximum.)

7. Let $p = 100 - \frac{q^2}{3}$ for $0 \le q \le 17$ be the price per unit at which q units of a certain commodity will sell.

a) Find the Revenue and Marginal Revenue functions as functions of the number of units sold.

6

$R(q) = \left(100 - \frac{q^2}{3}\right)q = \boxed{100q - \frac{q^3}{3}}$ Revenue. $R'(q) = 100 - \frac{3q^2}{3} = \boxed{100 - q^2}$ marginal revenue

b) For what level of production (that is, at what number of units sold) is revenue maximized?

3

let $100 - q^2 = 0$

$q^2 = 100$

$q = \pm 10$

+10 is the only value in the domain.

Is it a maximum? yes

$100-q^2$: 0 + 10 − 17

$\boxed{\text{at } q = 10 \text{ units, revenue is maximized}}$

c) If the total cost to produce q units is $C(q) = 36q + 25$ dollars, find the profit function and the marginal profit function.

4

$P(q) = 100q - \frac{q^3}{3} - (36q + 25) = 100q - \frac{q^3}{3} - 36q - 25 = \boxed{64q - \frac{q^3}{3} - 25 : \text{Profit}}$

$P'(q) = 64 - \frac{3q^2}{3} = \boxed{64 - q^2 : \text{Marginal profit}}$

d) For what level of production is profit maximized?

3

(just need q-value)

let $64 - q^2 = 0$

$(8-q)(8+q) = 0$

$q = 8$ $q = -8$, but only $q = 8$ is in domain.

Maximum? yes

$\boxed{\text{Maximum profit at } q = 8 \text{ units.}}$

	0	8	17
$8-q$	+		−
$8+q$	+		+
	+		−

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