

**Math 105, Precalculus**  
**Test 2, Chapter 2**

Name ANSWER KEY A  
 October 27, 2009

Please show all work neatly. Use of calculators is not permitted. Place your answers in the spaces provided or in boxes

1. Consider the following piecewise defined function:

$$f(x) = \begin{cases} -x^2 + 2, & \text{if } x > -1 \\ 3x + 6, & \text{if } x < -1 \\ 0, & \text{if } x = -1 \end{cases}$$

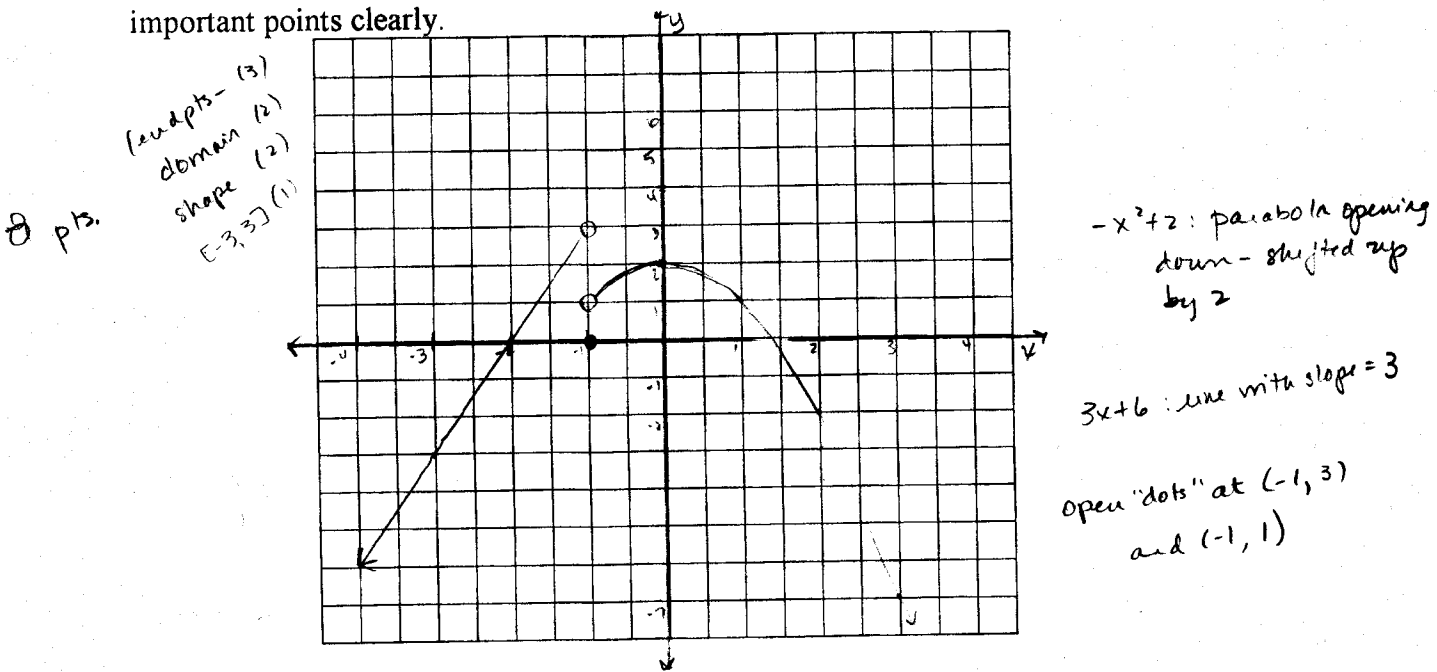
a) Find the following, showing all work.

i)  $f(-1)$  0

ii)  $f(-2)$  0  $(3)(-2) + 6 = -6 + 6 = 0$

iii)  $f(1)$  1  $-(1)^2 + 2 = -1 + 2 = 1$

b) Carefully graph the function  $f(x)$  on a domain of at least  $[-3, 3]$ . Label axes and important points clearly.

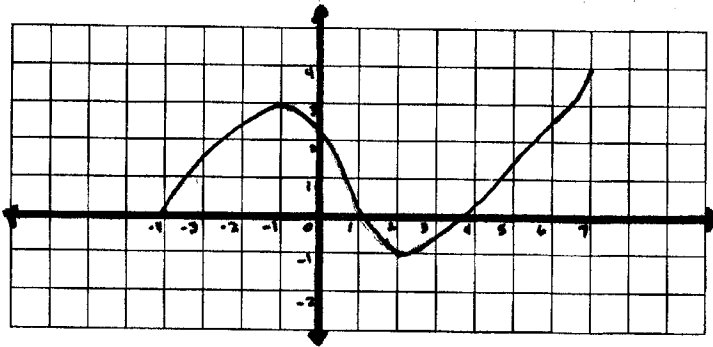


2. Let  $g(x) = x^2 - 3$ . Find the average rate of change of  $g(x)$  between the values  $x = -1$  and  $x = -1 + h$ . Reduce your answer as much as legally possible.

$$\frac{g(-1+h) - g(-1)}{-1+h - (-1)} = \frac{(-1+h)^2 - 3 - (-1^2 - 3)}{h} = \frac{1 - 2h + h^2 - 3 - (-1 - 3)}{h} = \frac{-2h + h^2}{h} = \frac{h(-2+h)}{h}$$

$-2 + h$

3. Consider the graph below.



16 pts.  
3 a) Does the graph represent a function? yes Why or why not? Passes vertical line test / only 1 y-value for each x-value

4 b) In interval notation, state the domain and range of this graph:

i) Domain  $[-4, 7]$

ii) Range  $[-1, 4]$

3 c) Is this graph one-to-one? No Why or why not? Fails horizontal line test (multiple values of x map to the same y.)

3 d) Find the average rate of change between  $x = -1$  and  $x = 2$ . Points:  $(-1, 3)$  and  $(2, -1)$   
Avg rate of change:  $\frac{3 - (-1)}{-1 - 2} = \frac{4}{-3} = \boxed{-\frac{4}{3}}$

4 e) Over what interval(s) is the graph increasing?  $[-4, -1)$  and  $(2, 7]$

4. Let  $f(x) = x^2 + 4x + 1$

a) Express the quadratic function  $f(x)$  in standard form:

$$f(x) = x^2 + 4x + 4 - 4 + 1 = \boxed{(x+2)^2 - 3}$$

$$b = 4$$

$$\frac{b}{2} = 2$$

$$\left(\frac{b}{2}\right)^2 = 4$$

b) Find the following:

3 i) Vertex (both coordinates)  $(-2, -3)$

3 ii) x-intercept(s), if any  $x = -2 \pm \sqrt{3}$

$$\text{let } (x+2)^2 - 3 = 0 \quad (x+2)^2 = 3 \quad x+2 = \pm\sqrt{3} \quad x = -2 \pm \sqrt{3}$$

2 iii) y-intercept  $y = 1$  (or  $(0, 1)$ )

2 iv) The function's maximum or minimum value, and identify it as a maximum or minimum minimum value =  $-3$  at  $x = -2$  (vertex)

c) Sentence completion: Compared to the graph of  $g(x) = x^2$ , the graph of

4  $f(x) = x^2 + 4x + 1$  is shifted horizontally by 2 units to the left (direction)

and vertically by 3 units up/down (choose one.)

5. The graph of the function  $f(x)$  is given below.

a) Match each of the following functions with its graph. (Place the letter of the appropriate graph in the space provided.)

i)  $g(x) = f(x) + 1$  F (shifted up by 1)

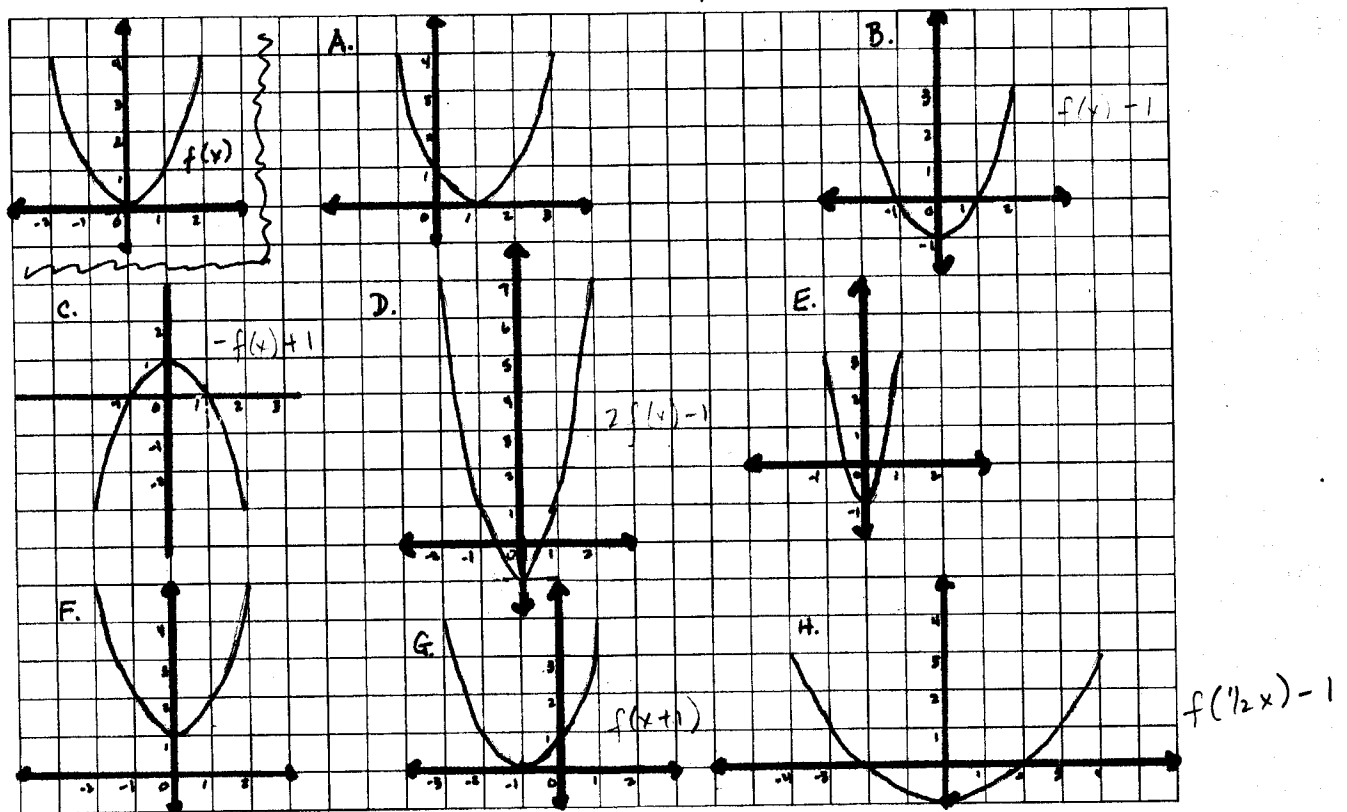
ii)  $h(x) = f(x-1)$  A (shifted right by 1)

iii)  $r(x) = f(2x) - 1$  E (horizontal shrink by  $1/2$ , shift down by 1)

b) Choose one of the graphs you did not identify for parts i through iii above, and give a formula for it similar to the formulas above. \_\_\_\_\_

c) List all graphs below, if there are any, that could be the graphs of even functions.

A, B, C, D, E, F, H (symmetric across y-axis) List all graphs below, if there are any, that could be the graphs of odd functions. None (none are symmetric across origin)



6. Let  $f(x) = \sqrt{x}$  and let  $g(x) = \frac{1}{x-5}$ . Find the following functions and the domain of each of

a)  $\frac{f}{g} = \frac{\sqrt{x}}{\frac{1}{x-5}} = (\sqrt{x})(x-5)$

Domain:  $\{x \geq 0, x \neq 5\}$

b)  $f \circ g \left( \frac{1}{x-5} \right) = \sqrt{\frac{1}{x-5}}$

Domain:  $\{x \mid x > 5\}$   $\frac{1}{x-5} > 0$

c)  $g \circ f \left( \frac{1}{\sqrt{x}} \right) = \frac{1}{\sqrt{x}-5}$

Domain:  $\{x \mid x \geq 0, x \neq 25\}$

Zero-denominator  
let  $\sqrt{x}-5=0$

$\sqrt{x} = 5$   
 $x = 25$

13 pts.

6

3

4 pts.

18 pts.

27 pts.  
31

7. Find functions  $f$  and  $g$  such that the function  $F(x) = (4x-7)^3$  can be expressed in the form  $f \circ g$ .

5 pts.

$$g = 4x - 7 \quad : f(g(x)) = f(4x-7) = (4x-7)^3 \checkmark$$

$$f = x^3$$

8. Let  $f(x) = x^2 - 5$ , for  $x \geq 0$ .

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a) Find the inverse function  $f^{-1}$ .

$$f^{-1}(y) = \sqrt{y+5}$$

or

$$f^{-1}(x) = \sqrt{x+5}$$

$$y = x^2 - 5$$

$$y + 5 = x^2$$

$$x = \sqrt{y+5}$$

- b) What is the domain of  $f^{-1}$ ?  $[-5, +\infty)$

2

(or  $y \geq -5$ )

- c) Use the Inverse Function Property to show that  $f(x)$  and the inverse function you found are inverses of each other by finding either  $(f \circ f^{-1})$  or  $(f^{-1} \circ f)$ . (You only need to compose in one direction.) Label clearly which way you are composing the functions.

4

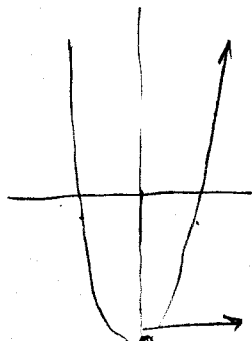
$$f \circ f^{-1} = f(\sqrt{y+5}) = (\sqrt{y+5})^2 - 5 = y + 5 - 5 = y \checkmark$$

$$f^{-1} \circ f = f^{-1}(x^2 - 5) = \sqrt{(x^2 - 5) + 5} = \sqrt{x^2} = x \checkmark$$

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- d) Extra credit: Why is the domain of  $f$  limited to  $\{x|x \geq 0\}$ ?

The domain of  $f$  must be restricted, so that  $f$  will be one-to-one function. (It is not one-to-one on its entire domain).  $x=0$  is the vertex; and  $f$  is one-to-one on either half of its domain that starts from the vertex.



-5 this half is 1-to-1

16