

Math 105, Precalculus
Test 2, Chapter 2

Name ANSWER KEY B
 October 27, 2009

Please show all work neatly. Use of calculators is not permitted. Place your answers in the spaces provided or in boxes

1. Consider the following piecewise defined function:

$$f(x) = \begin{cases} 4x - 5, & \text{if } x > 1 \\ -x^2 + 4, & \text{if } x < 1 \\ 0, & \text{if } x = 1 \end{cases}$$

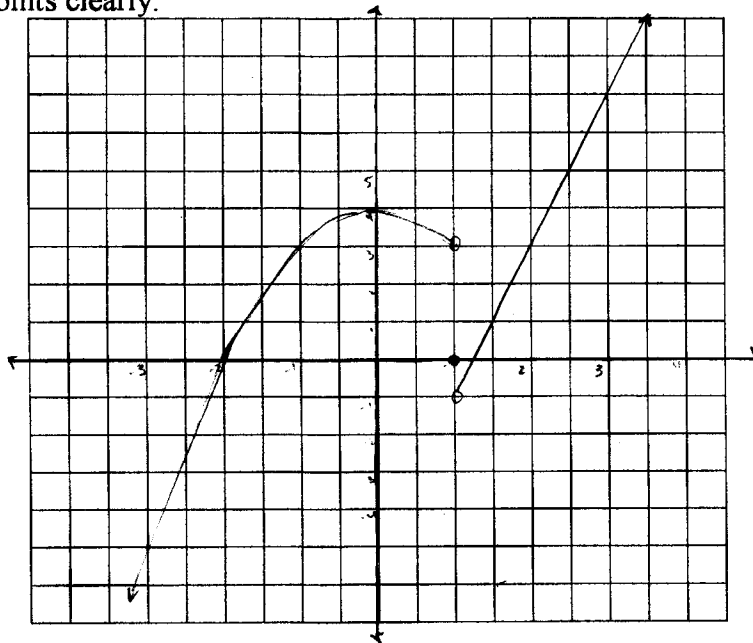
a) Find the following, showing all work.

2 i) $f(-1)$ 3 $-(-1)^2 + 4 = -1 + 4 = 3$

2 ii) $f(1)$ 0

2 iii) $f(2)$ 3 $4(2) - 5 = 8 - 5 = 3$

b) Carefully graph the function $f(x)$ on a domain of at least $[-3, 3]$. Label axes and important points clearly.



$4x - 5$: linear function,
 slope = 4.

8

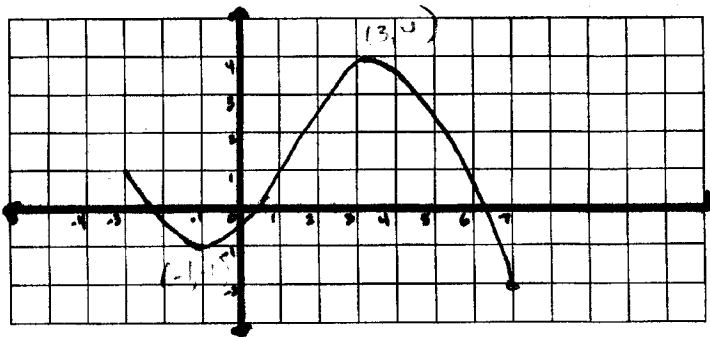
7 2. Let $g(x) = 2 - x^2$. Find the average rate of change of $g(x)$ between the values $x = 1$ and $x = 1 + h$. Reduce your answer as much as legally possible.

$$\frac{g(1+h) - g(1)}{1+h-1} = \frac{2 - (1+h)^2 - [2 - 1^2]}{h} = \frac{2 - (1+2h+h^2) - 2 + 1}{h} = \frac{2 - 1 - 2h - h^2 - 2 + 1}{h}$$

$$\frac{-h(-2-h)}{h} = \boxed{-2-h}$$

21

3. Consider the graph below.



3 a) Does the graph represent a function? yes Why or why not? Passes vertical line test.

4 b) In interval notation, state the domain and range of this graph:

i) Domain $[-3, 7]$

ii) Range $[-2, 4]$

3 c) Is this graph one-to-one? No Why or why not? Fails horizontal line test.

d) Find the average rate of change between $x = -1$ and $x = 3$. Points: $(3, 4), (-1, -1)$

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$$\frac{4 - (-1)}{3 - (-1)} = \frac{4 + 1}{3 + 1} = \frac{5}{4}$$

4 e) Over what interval(s) is the graph decreasing? $[-3, -1]$ and $[3, 7]$

4. Let $f(x) = x^2 + 8x + 9$.

a) Express the quadratic function $f(x)$ in standard form:

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$$f(x) = x^2 + 8x + 16 - 16 + 9 = (x + 4)^2 - 7$$
 $b = 8$ $(\frac{b}{2})^2 = 16$
 $\frac{b}{2} = 4$

b) Find the following:

3 i) Vertex (both coordinates) $(-4, -7)$

3 ii) x-intercept(s), if any $x = -4 \pm \sqrt{7}$

Let $(x + 4)^2 - 7 = 0$; $(x + 4)^2 = 7$; $x + 4 = \pm\sqrt{7}$; $x = -4 \pm \sqrt{7}$

2 iii) y-intercept $y = 9$ (or $(0, 9)$)

2 iv) The function's maximum or minimum value, and identify it as a maximum or minimum minimum value is -7 (at $x = -4$) - at vertex. No maximum.

4 c) Sentence completion: Compared to the graph of $g(x) = x^2$, the graph of

$f(x) = x^2 + 8x + 9$ is shifted horizontally by 4 units to the left (direction)

and vertically by 7 units up/down (choose one.)

5. The graph of the function $f(x)$ is given below.

a) Match each of the following functions with its graph. (Place the letter of the appropriate graph in the space provided.)

i) $g(x) = f(x+1)$ F (shift 1 to left)

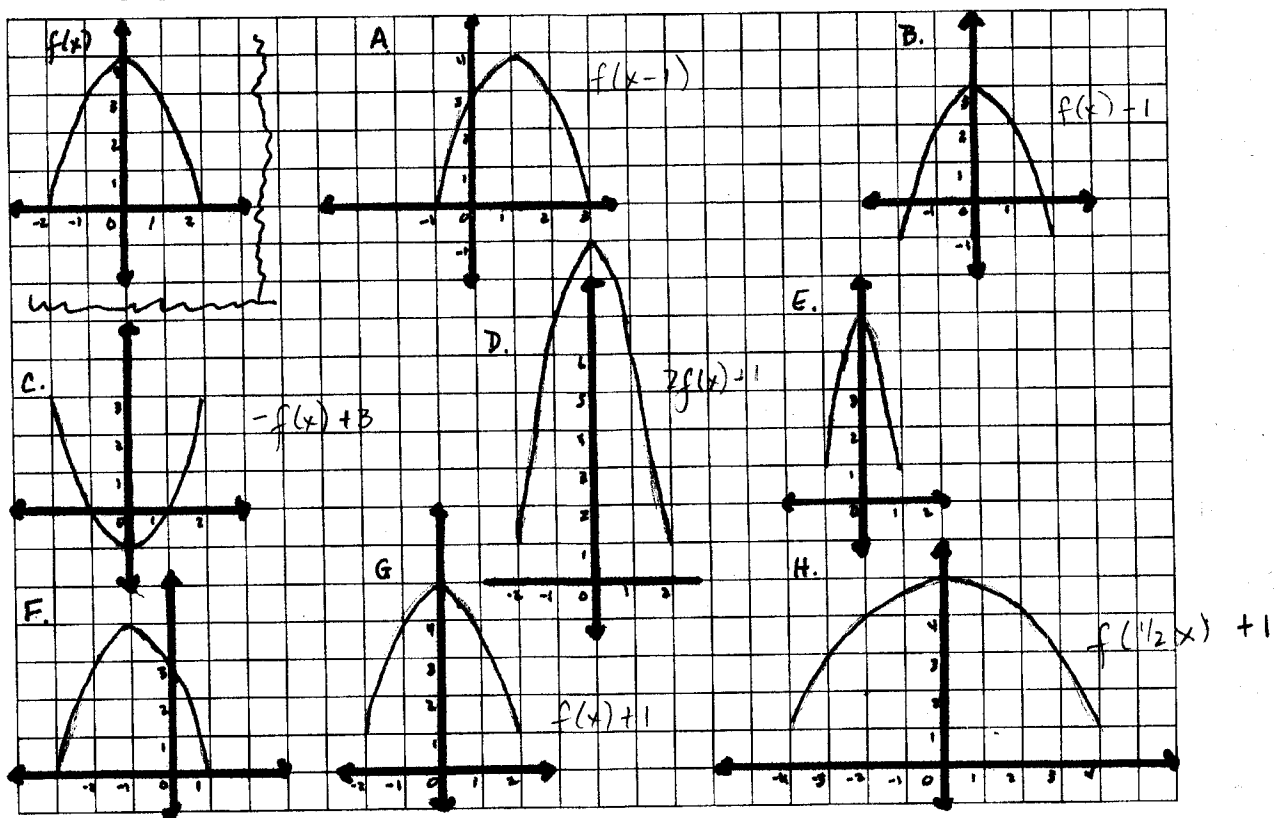
ii) $h(x) = f(x) - 1$ B (shift 1 down)

iii) $r(x) = f(2x) + 1$ E (horizontal shrink by $1/2$, up 1)

b) Choose one of the graphs you did not identify for parts i through iii above, and give a formula for it similar to the formulas above. _____

c) List all graphs below, if there are any, that could be the graphs of even functions.

d) B, C, D, E, G, H List all graphs below, if there are any, that could be the graphs of odd functions. None



18pts. 6. Let $f(x) = \frac{1}{x-4}$ and let $g(x) = \sqrt{x}$. Find the following functions and the domain of each of the new functions:

a) $\frac{f}{g} = \frac{\frac{1}{x-4}}{\sqrt{x}}$

Domain: $\{x > 0, x \neq 4\}$

b) $f \circ g = f(\sqrt{x}) = \frac{1}{\sqrt{x}-4}$

Domain: $\{x \geq 0, x \neq 16\}$

c) $g \circ f = \sqrt{\frac{1}{x-4}}$

Domain: $x > 4$

7. Find functions f and g such that the function $F(x) = \sqrt[4]{5x-2}$ can be expressed in the form $f \circ g$.

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$$g := 5x - 2$$

$$f := \sqrt[4]{x} = x^{1/4}$$

8. Let $f(x) = x^2 + 2$, for $x \geq 0$.

a) Find the inverse function f^{-1} .

$$f^{-1} = \sqrt{y-2}$$

$\text{or } f^{-1}(y) = \sqrt{y-2}$

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let $y = x^2 + 2$

$$y - 2 = x^2 \quad \sqrt{y-2} = x = f^{-1}$$

- b) What is the domain of f^{-1} ? $y \geq 2$ (or $[2, +\infty)$

2

- c) Use the Inverse Function Property to show that $f(x)$ and the function you found are inverses of each other by finding either $(f \circ f^{-1})$ or $(f^{-1} \circ f)$. (You only need to compose in one direction.) Label clearly which way you are composing the functions.

4

$$f \circ f^{-1} = f(\sqrt{y-2}) = (\sqrt{y-2})^2 + 2 = y - 2 + 2 = y \quad \checkmark$$

$$f^{-1} \circ f = f^{-1}(x^2 + 2) = \sqrt{(x^2 + 2) - 2} = \sqrt{x^2 + 0} = x \quad \checkmark$$

- d) Extra credit: Why is the domain of f limited to $\{x | x \geq 0\}$?

(see answer on other version)

16