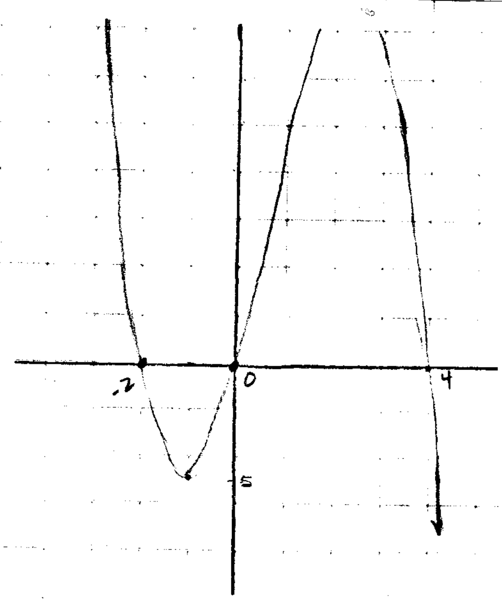
Math 105, Sample questions for Test 3 (Sections 3.1, 3.2, 3.6, 4.1-4.4), 11/~~24~~ 30/09
Fall 2009

1. Sketch the graph of the polynomial: $P(x) = -x^3 + 2x^2 + 8x$. Be sure to show and label all intercepts, and be certain that your graph demonstrates proper long-term behavior of the polynomial. $P(x) = -x(x^2-2x-8) = -x(x-4)(x+2)$

x-intercepts: let $-x=0$ $x-4=0$ $x+2=0$
$x=0$ $x=4$ $x=-2$

y-intercept $P(0) = -0^3 + 2\cdot 0^2 + 8\cdot 0 = 0$ $(0,0)$



Long term behavior:
$a=-1$ (leading coeff), degree $=3$
so $P(x) \to +\infty$ as $x \to -\infty$,
$P(x) \to -\infty$ as $x \to +\infty$.

test values:
$P(-3) = -(-27) + 2(9) + 8(-3)$
$= 27 + 18 - 24 = 21$
$P(-1) = -(-1) + 2(1) + 8(-1)$
$= 1 + 2 - 8 = -5$
$P(2) = -8 + 8 + 16 = 16$

2. Consider the polynomial: $P(x) = x^3 + 2x^2 - 19x + 12$.
a) Use any method to show that $x = 3$ is a zero of $P(x)$.

3 possible methods;
a) $P(3) = 3^3 + 2(3)^2 - 19(3) + 12 = 27 + 18 - 57 + 12 = 57 - 57 = 0$ ✓
b) Long division by factor $(x-3)$:

$$
\begin{array}{r}
x^2 + 5x - 4 \\
x-3 \overline{\smash{)}\, x^3 + 2x^2 - 19x + 12} \\
-(x^3 - 3x^2) \\
\hline
5x^2 - 19x \\
-(5x^2 - 15x) \\
\hline
-4x + 12 \\
-4x + 12 \\
\hline
0
\end{array}
$$

0 ← Zero Remainder

c) Synthetic division:

$$
\begin{array}{c|cccc}
3 & 1 & 2 & -19 & 12 \\
 & \downarrow & 3 & 15 & -12 \\
\hline
 & 1 & 5 & -4 & \boxed{0}
\end{array}
$$

↑ zero Remainder

b) Write $P(x)$ as a factored polynomial, with one term as the factor with the zero in part a) above and the other term a quadratic function. $P(x) = (x-3)(x^2+5x-4)$

c) Find all zeros of the polynomial $P(x)$.

Let $x^2 + 5x - 4 = 0$. Does not factor: use Quadratic formula:
$x = \dfrac{-5 \pm \sqrt{25 - 4(-4)(1)}}{2\cdot 1} = \dfrac{-5 \pm \sqrt{25+16}}{2} = \dfrac{-5 \pm \sqrt{41}}{2}$

So the zeros are $x = 3$, $x = \dfrac{-5+\sqrt{41}}{2}$, $x = \dfrac{-5-\sqrt{41}}{2}$.

3. Match each of the polynomials below with one of the graphs below and on the next page. Give a brief justification each of your answers, including information on x- and y- intercepts and end behavior of the function. Write possible polynomials for all the graphs you did not use.

a) $P(x) = -x(x+2)^2$ G

b) $R(x) = x^2(x-2)$ C

c) $P(x) = x(x-2)^2$ B

d) $R(x) = -x(x^2-4)$ F

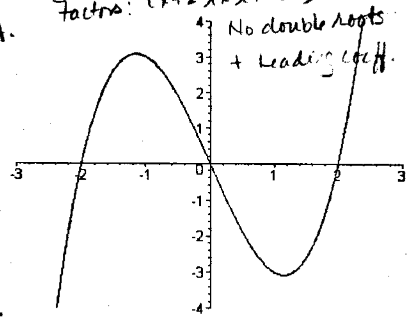
e) $R(x) = x^2(x-2)^2$ D

f) $P(x) = -x(x-2)^2$ J

g) $R(x) = -x^2(x-2)$ I

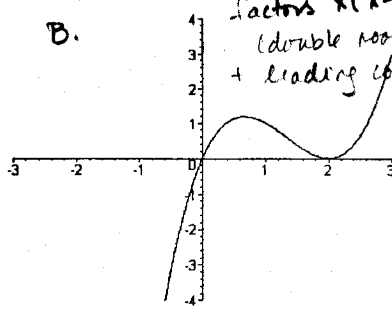
A.

Factors: $(x+2)(x)(x-2)$
No double roots
+ Leading coeff.



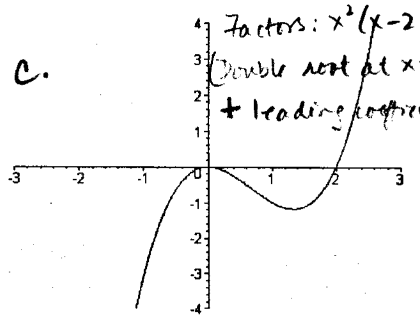
B.

Factors $x(x-2)^2$
(double root at $x=2$)
+ leading coeff.



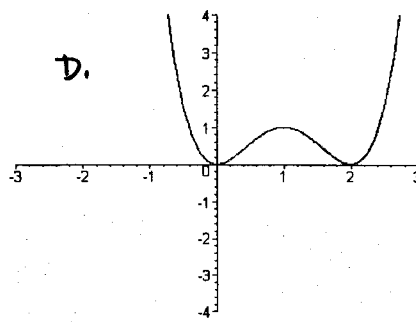
C.

Factors: $x^2(x-2)$
(Double root at $x=0$)
+ leading coefficient



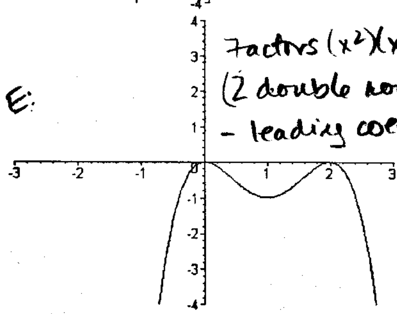
D.

factors $(x^2)(x-2)^2$
(2 double roots)
+ leading coefficient



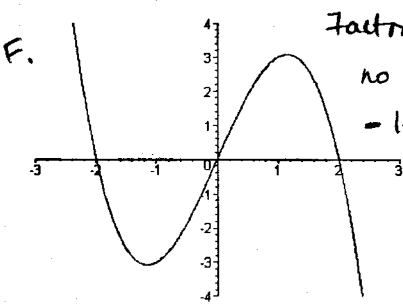
E.

Factors $(x^2)(x-2)^2$
(2 double roots)
− leading coefficient



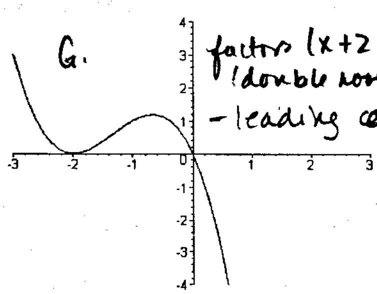
F.

Factors $(x+2)(x)(x-2)$
no double roots
− leading coefficient



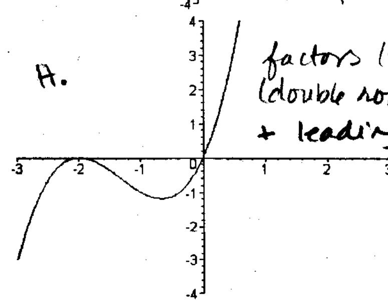
G.

factors $(x+2)^2(x)$
(double root at $x=-2$)
− leading coefficient



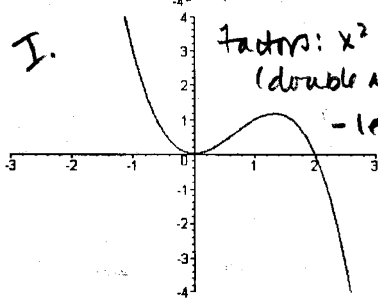
H.

factors $(x+2)^2(x)$
(double root at $x=-2$)
+ leading coefficient



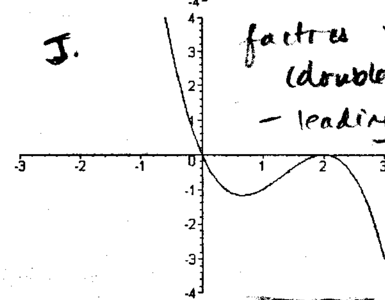
I.

factors: $x^2(x-2)$
(double root at $x=0$)
− leading coefficient



J.

factors $x(x-2)^2$
(double root at $x=2$)
− leading coefficient.



4. Find the quotient and remainder: $\dfrac{2x^3+6x+1}{x^2+2x}$

$$
\begin{array}{r}
2x - 4 + \dfrac{14x+1}{x^2+2x} \\
x^2+2x \,\big)\, 2x^3+0x^2+6x+1 \\
-(2x^3+4x^2) \\
\hline
-4x^2+6x \\
-(-4x^2-8x) \\
\hline
14x+1
\end{array}
$$

Quotient: $2x-4$

Remainder: $14x+1$

5. Let $g(x)=\dfrac{4x-8}{(x-4)(x+2)}$.

a) Find the following:

i) x-intercepts, if any $\boxed{(2,0)}$

Let $4x-8=0$; $4x=8$; $\boxed{x=2}$

ii) y-intercept, if any $\boxed{y=1 \quad (\text{or } (0,1))}$

Let $x=0$; $g(0)=\dfrac{4\cdot 0-8}{(0-4)(0+2)}=\dfrac{-8}{-8}=1$

iii) Domain $\{x \mid x\neq 4,\ x\neq -2\}$

Check for zero denominator $x=4,\ x=-2$

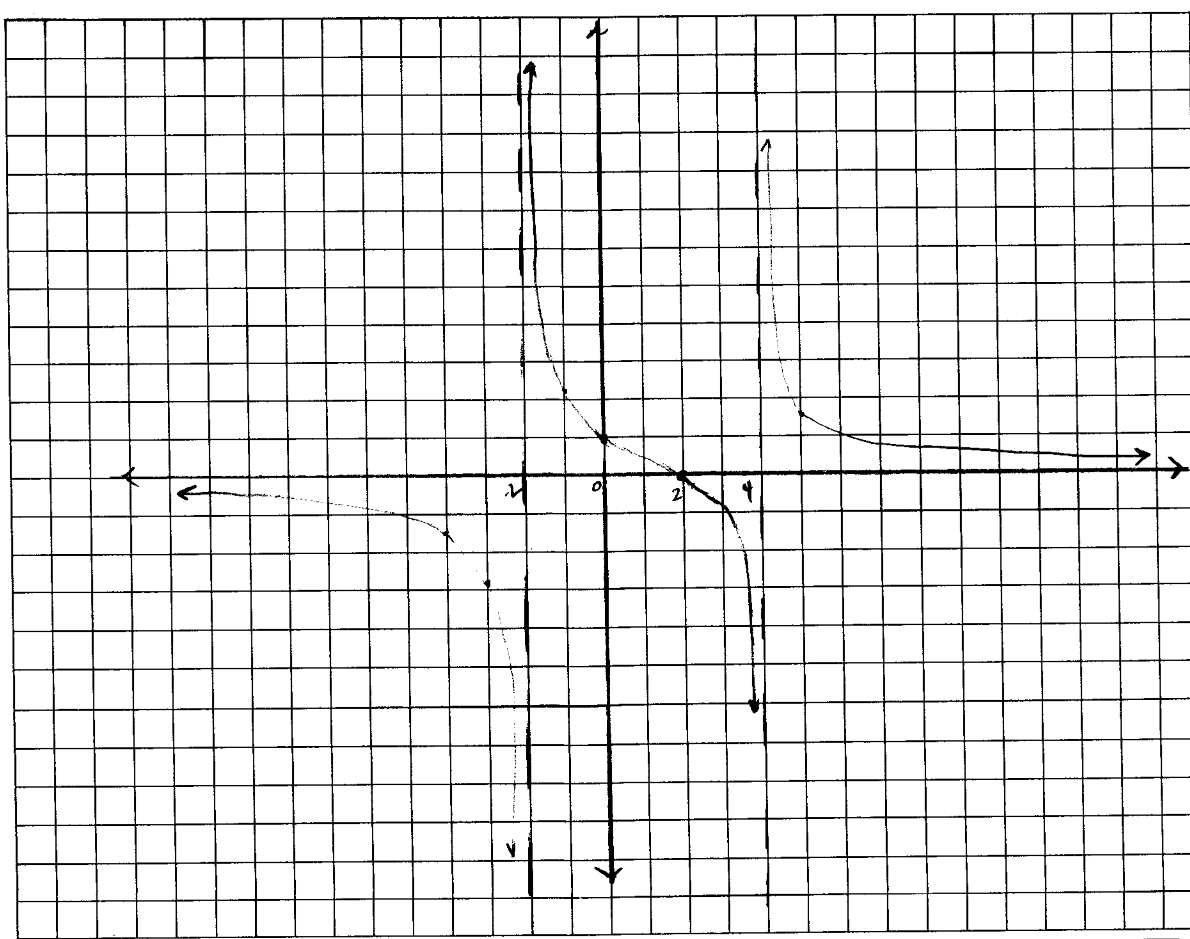
iv) Vertical asymptote(s), if any at $x=4$, $x=-2$ (at zeros in denominator of reduced function)

v) Horizontal asymptote, if any $y=0$

(degree of denominator > degree of numerator)

as $x\to\infty$ $\dfrac{4x-8}{x^2-2x-8}$ $\dfrac{\frac{4x}{x^2}-\frac{8}{x^2}}{\frac{x^2}{x^2}-\frac{2x}{x^2}-\frac{8}{x^2}}$ $=\dfrac{0}{1}=0$

b) Graph the function $g(x)$.



Test values

x	$g(x)$
3	$\frac{4}{-1(5)}$
5	$\frac{12}{7}$
-1	$\frac{-12}{(-5)(1)} = \frac{12}{5}$
-3	$\frac{-20}{(-7)(-1)} = \frac{-20}{7}$
-4	$\frac{-24}{(-8)(-2)} = \frac{-24}{16} = \frac{-3}{2}$

Domain of $\ln x$ is $x > 0$. So the quantity $x^2 - 2x > 0$.
$x(x-2) > 0$. Let $x = 0$, $x - 2 = 0$; zeros at $x = 0$, $x = 2$.
Check intervals:

	-1	1	3
x	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$

$+ \quad 0 \quad - \quad 2 \quad +$

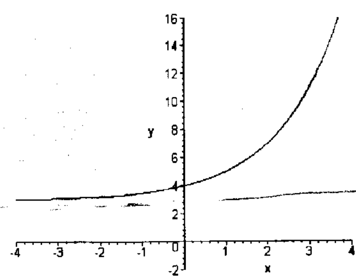
Domain   Domain

6. Find the domain of the functions:
   a) $h(x) = \ln(x^2 - 2x)$,
   $\therefore$ Domain is $\{x \mid x < 0 \text{ or } x > 2\}$ or $(-\infty, 0) \cup (2, +\infty)$
   b) $f(x) = \ln x + \ln(3 - x)$. Domain of $\ln x$ is $x > 0$. Domain of $\ln(3-x)$ is $3 - x > 0$, $3 > x$ or $x < 3$.
   Domain of the sum is $(0, +\infty) \cap (-\infty, 3) = (0, 3)$
   0   1   2   3   4
   intersection.
   c) $g(x) = e^{x^2 - 2x}$ Domain is $\mathbb{R}$
   (domain of exponential functions & of polynomials is $\mathbb{R}$)

7. Evaluate the following expressions. (Your answer in each case should be a number, and this question will be on the non-calculator portion of the test.)
   a) $\log_3 18 - \log_3 2 = \log_3 \frac{18}{2} = \log_3 9 = \log_3 3^2 = \boxed{2}$
   b) $e^{2\ln 3} = e^{\ln 3^2} = e^{\ln 9} = \boxed{9}$. or $e^{2\ln 3} = (e^{\ln 3})^2 = 3^2 = 9$
   c) $\log_4 \frac{1}{16} = \log_4 \frac{1}{4^2} = \log_4 4^{-2} = \boxed{-2}$

8. Rewrite the expression as a single logarithm: $4\ln x - 2\ln(x-2) + \ln(x^2 - 4) =$ single log
   $\ln x^4 - \ln(x-2)^2 + \ln(x^2 - 4) = \ln \frac{x^4(x^2-4)}{(x-2)^2} = \ln \frac{x^4 (x+2)(x-2)}{(x-2)^2} = \boxed{\ln \frac{x^4(x+2)}{(x-2)}}$

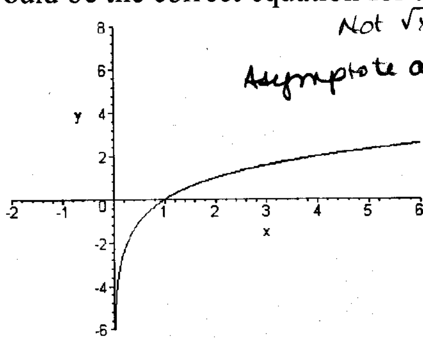
9. Which of the following equations would be the correct equation for the graph below?
   a) $f(x) = x^2 + 3$
   b) $g(x) = \log_2(x) + 3$
   c) $h(x) = 2^x + 3$
   d) $k(x) = 2^{x-3}$
   e) $n(x) = \log_2(x + 3)$



Must be an exponential function because it has a horizontal asymptote as $x \to -\infty$.
horizontal asymptote at $y = 3$, so shifted up by 3 (d would just be shifted right, so HA would still be $y = 0$).

Briefly justify your answer:

10. Which of the following equations would be the correct equation for the graph below?
   a) $f(x) = \sqrt{x}$
   b) $g(x) = 2^{-x}$
   c) $h(x) = -2^x$
   d) $k(x) = \log_2(x)$
   e) $n(x) = -\log_2(x)$



Not $\sqrt{x}$, because there is a vertical asymptote at $x = 0$. Not exponential (domain isn't $\mathbb{R}$). Must be logarithmic.
Not (e), because not reflected across x-axis.

Briefly justify your answer:

11. Use the laws of logarithms to rewrite the expression below in a form with no logarithms of a product, quotient or power:

$$\log\left(\frac{(x^2+1)\sqrt{x-1}}{x^3}\right) = \log(x^2+1) + \log\sqrt{x-1} - \log x^3$$
$$= \log(x^2+1) + \log(x-1)^{1/2} - 3\log x$$
$$= \boxed{\log(x^2+1) + \frac{1}{2}\log(x-1) - 3\log x}$$

12. Solve for x: (do not use a calculator to solve these problems; these problems would be on the non-calculator portion of the test.)

a) $2^{3x-4}+3=7$; $2^{3x-4}=4=2^2$; $3x-4=2$; $3x=6$; $\boxed{x=2}$

b) $\log_8(x-3)+\log_8(2)=1$ rewrite exponentially

$\log_8(x-3)(2)=1$; $8^1=(x-3)(2)=2x-6$; $2x=14$; $\boxed{x=7}$

c) $2\ln x=\ln 2+\ln(x+4)$

$\ln x^2=\ln 2(x+4)$; $x^2=2x+8$; $x^2-2x-8=0$; $(x-4)(x+2)=0$

$\boxed{x=4}$ $\cancel{x=-2}$ Not in domain

d) $\log_9 x=-\frac{1}{2}$ $9^{-1/2}=x=\frac{1}{\sqrt{9}}=\boxed{\frac{1}{3}}$

13. TRUE/FALSE. Justify your answer. If the statement is false, write a true statement.

F a) $\ln(2+e)=(\ln 2)(1)=\ln 2$ $\ln(2+e)\neq\ln 2+\ln e$ (can't split apart a sum) F

$\ln(2\cdot e)=\ln 2+\ln e=\ln 2+1$: True. or $\ln(2+e)$ does not simplify

F b) $\ln(3e^2)-\frac{2}{3}e^{\ln 3}=3$ F

$\ln 3+\ln e^2-\frac{2}{3}(3)=\ln 3+2-2=\ln 3\neq 3$

True statement: $\ln(3e^2)-\frac{2}{3}e^{\ln 3}=\ln 3$.

Solve each of the following problems as far as possible without a calculator. You should end up with exact answers for each (ie, with logs or exponentials). Use a calculator only to get decimal approximations of your final answers.

14. Sally has \$3,000 saved from her summer earnings, which she wants to invest for 2 years.

a) Which of the following is the best investment? (Circle the correct answer and show the amount she will have at the end of 2 years for each investment strategy.)

i) 3 %, compounded annually $3000(1+.03)^2=$ \$3182.70 ← BEST

ii) 2.95 %, compounded monthly $3000\left(1+\frac{.0295}{12}\right)^{12\cdot 2}=$ \$3182.10

iii) 2.9 %, compounded continuously $3000\cdot e^{.029\cdot 2}=$ \$3179.14

b) How long would it take Sally to double her original \$3000 investment, if she invests in the bank that pays 2.9%, compounded continuously? $\frac{6000}{3000}=\frac{3000\cdot e^{.029t}}{3000}$; $2=e^{.029t}$

$\ln 2=\ln e^{.029t}=.029t$; $t=\frac{\ln 2}{.029}$ exact answer $=23.9$ years

15. The number of snakehead fish in the Chesapeake Bay can be modeled by the function $P(t)=12\cdot(1.2)^t$, where t is the number of years after the year 2000.

a) What was the population of snakeheads in 2000? $(t=0)$ $12\cdot(1.2)^0=12\cdot 1=$ $\boxed{12 \text{ snakeheads}}$

b) Find the projected population of snakeheads in the year 2008. $\boxed{12\cdot(1.2)^8}$ exact answer $=51.5\sim 52$ snakeheads

c) After how many years is it projected that the population of snakeheads will reach 300?

$300=12\cdot(1.2)^t$; $\frac{300}{12}=(1.2)^t$; $\log\left(\frac{300}{12}\right)=\log 1.2^t=t\log 1.2$; $t=\frac{\log\left(\frac{300}{12}\right)}{\log(1.2)}=\frac{\log 25}{\log 1.2}$ Exact answer $\doteq 17.65\sim 17.7$ years.

16. Solve the equation $\frac{21}{2+(6^x)}=3$ for x. Then use a calculator to find the solution correct to 4 decimal places.

$\frac{21}{3}=\frac{3(2+6^x)}{3}$; $7=2+6^x$; $5=6^x$. $\ln 5=\ln 6^x=x\ln 6$;

$\boxed{x=\frac{\ln 5}{\ln 6}=\frac{\log 5}{\log 6}}$ exact answer

$\doteq .89824\sim .8982$