MATH 105 Homework 3 ANSWER KEY

1. Section 3.2.

6.

$$
\begin{array}{r}
2x^3 + 4x^2 + 8 \\
x^2+0x-2 \,\big)\, 2x^5 + 4x^4 - 4x^3 + 0x^2 - x - 3 \\
-\left(2x^5 + 0 - 4x^3\right) \\
\hline
4x^4 + 0 + 0 \\
-\left(4x^4 + 0 - 8x^2\right) \\
\hline
8x^2 - x - 3 \\
-\left(8x^2 + 0 - 16\right) \\
\hline
-x + 13
\end{array}
$$

(NOTE: 0's inserted to account for all exponents.)

2 pts.

$$\boxed{\therefore\ 2x^5+4x^4-4x^3-x-3 = \underbrace{(x^2-2)}_{D(x)}\underbrace{(2x^3+4x^2+8)}_{Q(x)} + \underbrace{(-x+13)}_{R(x)}}$$

(Note: this one can NOT be done with synthetic division)

18.

$$
\begin{array}{r}
3x^2 - 8x - 1 \\
x^2+x+3 \,\big)\, 3x^4 - 5x^3 + 0x^2 - 20x - 5 \\
-\left(3x^4 + 3x^3 + 9x^2\right) \\
\hline
-8x^3 - 9x^2 - 20x \\
-\left(-8x^3 - 8x^2 - 24x\right) \\
\hline
-x^2 + 4x - 5 \\
-\left(-x^2 - x - 3\right) \\
\hline
5x - 2
\end{array}
$$

($0x^2$ inserted to keep columns straight)

2 pts.

$$\boxed{\therefore\ \frac{3x^4-5x^3-20x-5}{x^2+x+3} = (3x^2-8x-1) + \frac{5x-2}{x^2+x+3} \qquad Q(x) + \frac{R(x)}{D(x)}}$$

- On these 2 problems, just take off ½ point for each error, up to 2 pts. for the entire problem.
- If they solve correctly with no "place-holder" zeros, don't take anything off. But if no place holders gets them in trouble, take off 1 point
- Don't take off points if answers aren't exactly in the form on this sheet (the form the book asks for), but you may add ½ pt per problem for getting the form exactly right.

To show that the values are 0's, it is sufficient to divide & get a 0 remainder (so parts a & b may be combined).

- they should get to quadratic (2 terms) in b and list 0's in c (should list all 4, but no points off if they forget)

2. $P(x) = 6x^4 - 7x^3 - 12x^2 + 3x + 2$, $c = -1$, $c = 2$

4 points

a) Show $c = -1$ is a zero: A. plug in: $P(-1) = 6(-1)^4 - 7(-1)^3 - 12(-1)^2 + 3(-1) + 2$
$= 6 + 7 - 12 - 3 + 2 = 0$ ✓

B. OR DIVIDE BY $(x+1)$ to check that the remainder $= 0$.

$$\begin{array}{r} 6x^3 - 13x^2 + x + 2 \\ x+1 \,\overline{)\,6x^4 - 7x^3 - 12x^2 + 3x + 2} \\ -(6x^4 + 6x^3) \\ \hline -13x^3 - 12x^2 \\ -(-13x^3 - 13x^2) \\ \hline x^2 + 3x \\ -(x^2 + x) \\ \hline +2x + 2 \\ 2x + 2 \\ \hline 0 \end{array}$$

$0 \leftarrow$ Remainder $= 0$; $\therefore c = -1$ is a zero.

$\therefore 6x^4 - 7x^3 - 12x^2 + 3x + 2 = (x+1)(6x^3 - 13x^2 + x + 2)$

(could also have been done with synthetic division.)

1pt. for each value (showing it's a zero & dividing)

Show $c = 2$ is a zero: A. PLUG IN (may plug in to reduced part):
$P(2) = (2+1)\cdot(6\cdot 2^3 - 13\cdot 2^2 + 2 + 2)$
$= 3(48 - 52 + 4) = 0$ ✓

OR B. Divide

$$\begin{array}{r|rrrr} 2 & 6 & -13 & 1 & 2 \\ & & 12 & -2 & -2 \\ \hline & 6 & -1 & -1 & \boxed{0} \end{array}$$

$\leftarrow$ Zero remainder ✓

(could also be done by long division.)

$\therefore 6x^4 - 7x^3 - 12x^2 + 3x + 2 = \underline{(x+1)(x-2)(6x^2 - x - 1)}$,

ANSWER to b)

1pt (have to see that the polynomial has been factored.)

c) Zeros at $\boxed{x = -1,\ x = 2}$

$6x^2 - x - 1 = 0$
$(3x+1)(2x-1) = 0$
$3x + 1 = 0 \qquad 2x - 1 = 0$
$3x = -1 \qquad 2x = 1$
$\boxed{x = -1/3 \qquad x = 1/2}$

1pt.

(4 zeros)
(consistent with degree of polynomial)

(may use quadratic formula, of course)

ii) $P(x) = x^4 - x^3 - 5x^2 + 3x + 6$; $c = -1$, $c = 2$

4pts like 2ii

$P(-1) = (-1)^4 - (-1)^3 - 5(-1)^2 + 3(-1) + 6$

$= 1 + 1 - 5 - 3 + 6 = 0$ ✓

Divide:

$$\begin{array}{r|l} & x^3 - 2x^2 - 3x + 6 \\ \hline x+1 & x^4 - x^3 - 5x^2 + 3x + 6 \\ & -(x^4 + x^3) \\ & \quad -2x^3 - 5x^2 \\ & \quad -(-2x^3 - 2x^2) \\ & \qquad -3x^2 + 3x \\ & \qquad -(-3x^2 - 3x) \\ & \qquad\quad +6x + 6 \\ & \qquad\quad +6x + 6 \\ & \qquad\quad \boxed{0} \end{array}$$

, so $P(x) = (x+1)(x^3 - 2x^2 - 3x + 6)$

✓ (Zero Remainder)

$\therefore c = -1$ is a zero.

Check $c = 2$: $P(2) = (2+1)(2^3 - 2(2)^2 - 3(2) + 6) = 3(8 - 8 - 6 + 6) = 0$ ✓

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -3 & 6 \\ & & 2 & 0 & -6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

b) $P(x) = (x+1)(x-2)(x^2-3)$

c) Zeros: $\boxed{x = -1, x = 2}$, $x^2 - 3 = 0$

$\boxed{x = \pm\sqrt{3}}$

4pts. 3. $P(x) = x^3 - x^2 - 8x + 12$; $c = 2$

1pt. $P(2) = 8 - 4 - 16 + 12 = 0$ ✓

or Divide

$$\begin{array}{r|l} & x^2 + x - 6 \\ \hline x-2 & x^3 - x^2 - 8x + 12 \\ & -(x^3 - 2x^2) \\ & \quad x^2 - 8x \\ & \quad -(x^2 - 2x) \\ & \qquad -6x + 12 \\ & \qquad \underline{-6x + 12} \\ & \qquad\quad 0 \end{array}$$

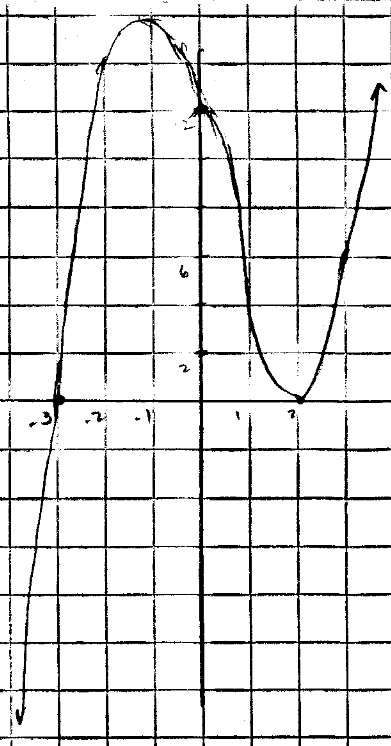
b) $\therefore P(x) = (x-2)(x^2 + x - 6)$

1pt. $= (x-2)(x+3)(x-2)$

$= (x-2)^2(x+3)$

1pt. c) Zeros at $x = 2$, $x = -3$ (double root at $x = 2$)

d) Graph:



x-intercepts: $x=2$, $x=-3$

y-intercept: $P(0)= 0+12 = 12$

Shape: cubic with positive leading coefficient

Zeros: graph crosses x-axis at $x=-3$, "bounces" at $x=2$

1pt.

Test values

x	$P(x)$
-4	$-64-16+32+12 = -80+44 = -36$
3	$27-9-24+12 = 6$

ii) $P(x) = x^3-7x^2+14x-8$, $c=1$. synthetic division

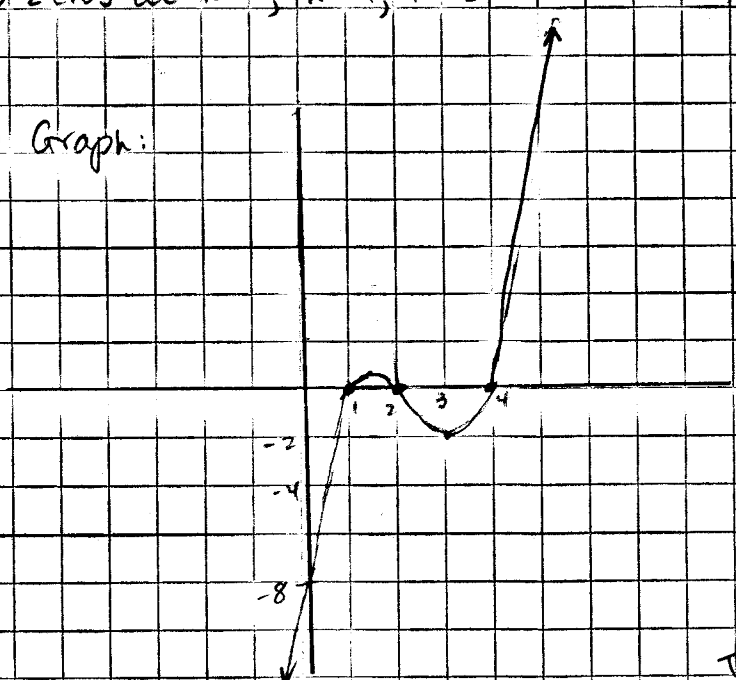
$$\begin{array}{r|rrrr} 1 & 1 & -7 & 14 & -8 \\ & & 1 & -6 & 8 \\ \hline & 1 & -6 & 8 & \boxed{0} \end{array}$$

$\leftarrow \therefore c=1$ is a zero

b) $= (x-1)(x^2-6x+8)$
$= (x-1)(x-4)(x-2)$

c) zeros at $x=1$, $x=4$, $x=2$

d) Graph:



x-intercepts: $x=1$, $x=2$, $x=4$

y-intercept: $P(0) = -8$

shape: cubic polynomial with positive leading coefficient, crosses x-axis at all 3 zeros

Test values

x	$P(x)$
$3/2$	$\frac{27}{8} - \frac{63}{4} + \frac{42}{2} - 8$
	$= \frac{27}{8} - \frac{126}{8} + \frac{168}{8} - \frac{64}{8} = 3/8$
3	$27-63+42-8 = -2$
5	$125-7(25)+70-8 = 12$

shapes of graphs may be very approximate, as long as end behavior & intercepts are right (∴ intervals are above or below x-axis)

(No points off if test values are missing)