Math 105-002 Precalculus
Test 3, Chapters 3 & 4

Name ANSWER KEY B
December 3, 2009

Use of calculators is **not** permitted on this test. You must show all work completely and neatly to receive credit for your answers.

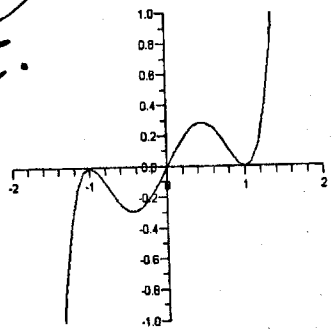
1. Match each of the polynomials below with one of the graphs. Give a very brief justification for each of your answers, including information on x- and y- intercepts, multiplicity of roots, or end behavior of the functions.

a) $f(x) = -x^6 + x^4$ IV $= -x^4(x^2-1) = -x^4(x+1)(x-1)$ even degree, negative leading coeff (opens down), even root at $x=0$, crosses axis at $x=1, x=-1$

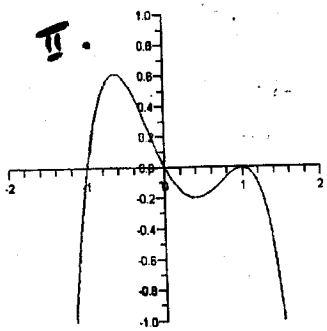
b) $g(x) = x^5 - x^3$ III $= x^3(x^2-1) = x^3(x+1)(x-1)$ odd degree, + l.c. (so as $x \to \infty$, $g(x) \to +\infty$; as $x \to -\infty$, $g(x) \to -\infty$) crosses x-axis at all roots,

c) $h(x) = -x(x-1)^2(x+1)^2$ VIII odd degree, negative l.c. (as $x \to +\infty$, $h(x) \to -\infty$; as $x \to -\infty$, $h(x) \to +\infty$) double roots at $x = \pm 1$, crosses x-axis at $x=0$.

I.



II.



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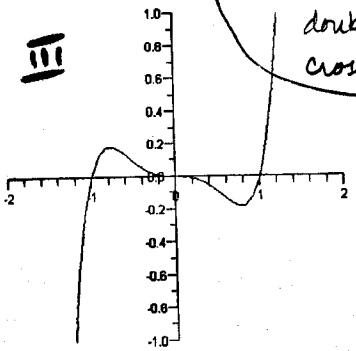

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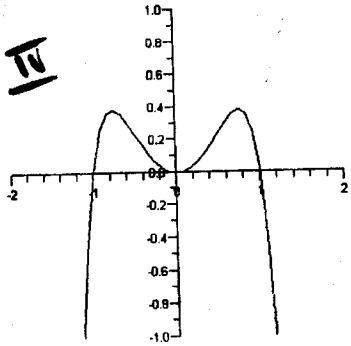

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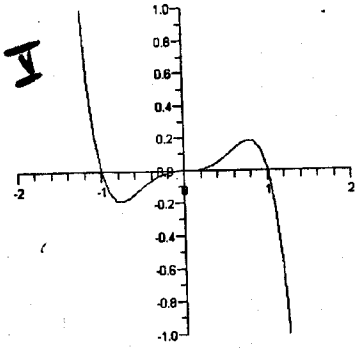

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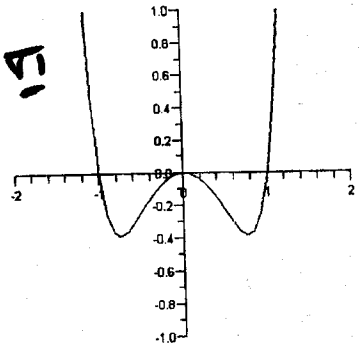
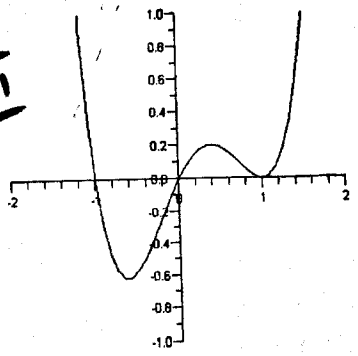
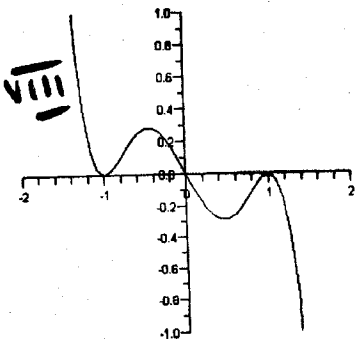


VII



VIII



2. Consider the polynomial: $P(x) = x^3 + 6x^2 + 8x + 3$.

a) Use any method to determine whether $x = 1$ and $x = -1$ are zeros of $P(x)$. Is $x = 1$ a zero? No Is $x = -1$ a zero? yes Be sure to show your work.

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$P(1) = 1^3 + 6(1)^2 + 8(1) + 3 = 18 \neq 0$

$P(-1) = (-1)^3 + 6(-1)^2 + 8(-1) + 3 = 9 - 9 = 0$ ✓

$x+1 \;\overline{)\; x^3 + 6x^2 + 8x + 3}$

$-(x^3 + x^2)$

$5x^2 + 8x$

$-(5x^2 + 5x)$

$3x + 3$

$-(3x + 3)$

0

$x-1 \;\overline{)\; x^3 + 6x^2 + 8x + 3}$

$x^2 + 7x + 15$

$-(x^3 - x^2)$

$7x^2 + 8x$

$-(7x^2 - 7x)$

$15x + 3$

$-(15x - 15)$

18 ← remainder $\neq 0$.

b) Write $P(x)$ as a factored polynomial, with one term including a zero you know and the other term a quadratic function.

4

$P(x) = (x+1)(x^2 + 5x + 3)$

	1	6	8	3
		1	7	15
	1	7	15	18

$\neq 0$

	1	6	8	3
−1		−1	−5	−3
	1	5	3	0

c) Find all zeros of the polynomial $P(x)$.

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let $x^2 + 5x + 3 = 0$

$x = \dfrac{-5 \pm \sqrt{25 - 4(1)(3)}}{2} = \dfrac{-5 \pm \sqrt{25 - 12}}{2}$

$= \dfrac{-5 \pm \sqrt{13}}{2}$

Zeros are $x = -1$, $x = \dfrac{-5 + \sqrt{13}}{2}$, $x = \dfrac{-5 - \sqrt{13}}{2}$

3. Find the quotient and remainder: $\dfrac{4x^3 + 5x^2 + 2}{x^2 - 1}$

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$x^2 + 0x - 1 \;\overline{)\; 4x^3 + 5x^2 + 0x + 2}$

$4x + 5$

$-(4x^3 + 0 - 4x)$

$5x^2 + 4x + 2$

$-(5x^2 + 0 - 5)$

$4x + 7$

Quotient : $4x + 5$

Remainder : $4x + 7$

$\dfrac{4x + 7}{x^2 - 1}$

4. Let $g(x)=\dfrac{3x+6}{x^2-2x-3}$. Find the following:

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✓ a) x-intercepts, if any $x=-2$ (or $(-2,0)$) Let $3x+6=0$; $3x=-6$; $x=-2$

✓ b) y-intercept, if any $y=-2$ (or $(0,-2)$) let $x=0$; $g(0)=\dfrac{0+6}{0-3}=-2$

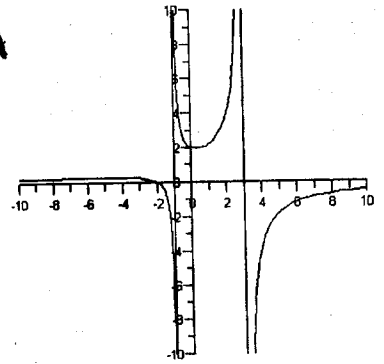
✓ c) Domain $\{x \mid x \neq 3, x \neq -1\}$ Let $x^2-2x-3=0=(x-3)(x+1)$; $x=3, x=-1$

✓ d) Vertical asymptote(s), if any $x=3$, $x=-1$

✓ e) Horizontal asymptote, if any $y=0$ (because degree of denominator > degree of numerator)

4 f) Which of the following could be the graph of $g(x)$? C Briefly justify your answer.

A



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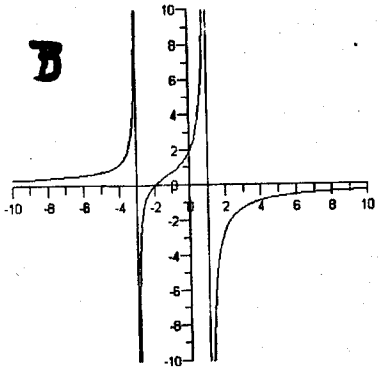

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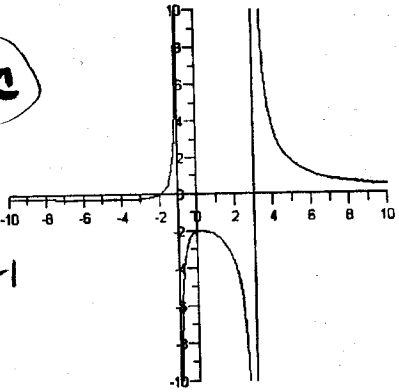

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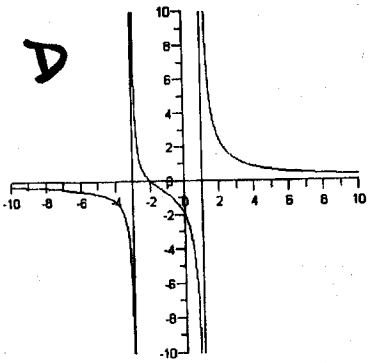

y-intercept = -2

x-intercept = -2

VA's at $x=3, x=-1$

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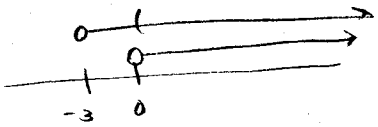
Note: Take the functions as they're given for domain.

5. Find the domain of each of the functions below

a) $h(x)=\ln(x)+\ln(x+3)$ $x>0$

$x+3>0$; $x>-3$



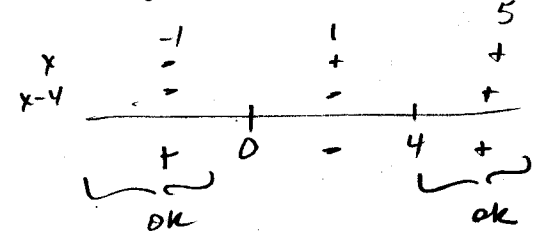
Intersection: $\{x \mid x>0\}$

b) $f(x)=x^3e^{5x^4}$

$\mathbb{R}$ (exponential and power functions)

c) $g(x)=\ln(x(x-4))$

$\{x \mid x<0 \text{ or } x>4\}$



6. True or False. Indicate whether each of the following statements is true or false. You must show work justifying your answer in order to receive full credit.

16 ✓ 3 TF 1 work

a) T $\log 50 + \log 20 = 3$

$\log 50 \cdot 20 = \log 1000 = \log 10^3 = 3$ ✓

b) F $\ln(3) \cdot \ln(7) = \ln(21)$ $\ln(21) = \ln(3 \cdot 7) = \ln 3 + \ln 7$

c) F $3^{2\log_3 5} = 10$ $= 3^{\log_3 5^2} = 3^{\log_3 25} = 25 \neq 10$

d) T $\log_2 12 = \frac{\ln 12}{\ln 2}$ (change of base)

7. Use the laws of logarithms to rewrite the expression below in a form with no logarithms of a product, quotient or power:

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$$\ln\left(\frac{e^3 x^2}{(z^3-2)^4}\right) = \ln e^3 + \ln x^2 - \ln(z^3-2)^4$$

$$= 3 + 2\ln x - 4\ln(z^3-2)$$

8. Solve for x. Where necessary, leave your answer in exact form.

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a) $\log(x+5) - \log(4) = 1$ 4

$\log\left(\frac{x+5}{4}\right) = 1$

$10^1 = \frac{x+5}{4} \Rightarrow 40 = x+5$ $\boxed{x = 35}$

b) $2\ln x = \ln 4 + \ln(x+3)$ 4

$\ln x^2 = \ln 4(x+3) = \ln(4x+12)$

$x^2 = 4x+12$ $x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$ $\boxed{x = 6}$ ~~$x = -2$~~

4 c) $\frac{20}{1+e^{-x}} = 4$

$20 = 4(1+e^{-x})$

$5 = 1+e^{-x}$

$4 = e^{-x}$ $\ln 4 = \ln e^{-x} = -x$

$\boxed{x = -\ln 4 = \ln 1/4}$