Math 105-002 Precalculus
Test 3, Chapters 3 & 4

Name ANSWER KEY A
December 3, 2009

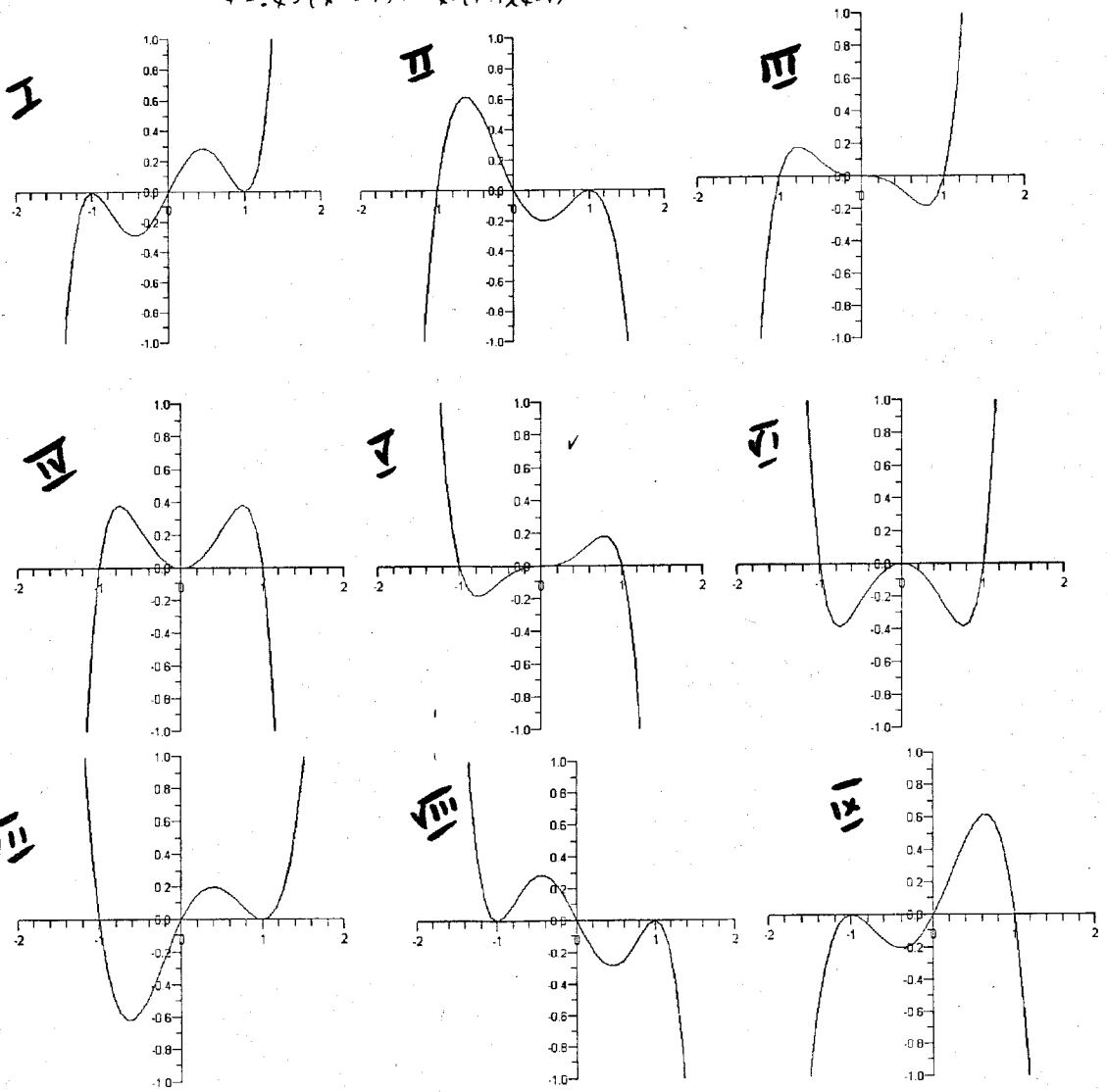
Use of calculators is **not** permitted on this test. You must show all work completely and neatly to receive credit for your answers.

1. Match each of the polynomials below with one of the graphs. Give a very brief justification for each of your answers, including information on x- and y- intercepts, multiplicity of roots, or end behavior of the functions.

   a) $f(x) = x^6 - x^4$ VI $= x^4(x^2-1) = x^4(x+1)(x-1)$. Zeros at $1, -1, 0$. "Bounces" at $x=0$. Degree is even, LC + $\Rightarrow$ opens up. U

   b) $g(x) = -x(x+1)(x-1)^2$ II Degree even, opens down, double root at $x=1$ only. (crosses x-axis)

   c) $h(x) = -x^5 + x^3$ V Degree odd, neg leading coeff. Crosses at all zeros.
   $= -x^3(x^2-1) = -x^3(x+1)(x-1)$



Wrong double root - even opens up

17

2. Consider the polynomial: $P(x) = x^3 + 8x^2 + 9x + 2$.

8 2 eq. 2 work

a) Use any method to determine whether $x = 1$ and $x = -1$ are zeros of $P(x)$. Is $x = 1$ a zero? No Is $x = -1$ a zero? yes Be sure to show your work.

Plugin: $P(1) = 1 + 8 + 9 + 2 = 20 \neq 0$

$P(-1) = -1 + 8 - 9 + 2 = 0$ ✓

or Divide $x-1 \overline{)\, x^3+8x^2+9x+2}$

$-(x^3 - x^2)$

$9x^2 + 9x$

$-(9x^2 - 9x)$

$18x + 2$

$-(18x - 18)$

$20 = R$

$x+1 \overline{)\, x^3+8x^2+9x+2}$

$-(x^3 + x^2)$

$7x^2 + 9x$

$-(7x^2 + 7x)$

$2x + 2$

$-(2x + 2)$

0

4

b) Write $P(x)$ as a factored polynomial, with one term including a zero you know and the other term a quadratic function.

$P(x) = (x+1)(x^2+7x+2)$

2 2

OR Synthetic Division

$1 \mid 1 \;\; 8 \;\; 9 \;\; 2$

$\downarrow \;\; 1 \;\; 9 \;\; 18$

$1 \;\; 9 \;\; 18 \;\; \boxed{20}$

$\neq 0$

$-1 \mid 1 \;\; 8 \;\; 9 \;\; 2$

$-1 \;\; -7 \;\; -2$

$1 \;\; 7 \;\; 2 \;\; \boxed{0}$

5

c) Find all zeros of the polynomial $P(x)$.

Let $x^2+7x+2=0$ use quadratic formula: $x = \dfrac{-7 \pm \sqrt{7^2 - 4(1)(2)}}{2 \cdot 1} = \dfrac{-7 \pm \sqrt{49-8}}{2}$

$= \dfrac{-7 \pm \sqrt{41}}{2}$

Zeros are $x = -1$, $x = \dfrac{-7+\sqrt{41}}{2}$, $x = \dfrac{-7-\sqrt{41}}{2}$

3. Find the quotient and remainder: $\dfrac{5x^3 + 2x^2 + 3}{x^2 - 1} = 5x + 2 + \dfrac{5x+5}{x^2-1}$

6

$$\begin{array}{r} 5x + 2 \\ x^2+0x-1 \;\overline{)\; 5x^3 + 2x^2 + 0x + 3} \\ -(5x^3 \quad + 0 \quad - 5x) \\ \hline 2x^2 + 5x + 3 \\ -(2x^2 \quad + 0 \quad - 2) \\ \hline 5x + 5 \end{array}$$

Quotient: $5x+2$

Remainder: $\dfrac{5x+5}{x^2-1}$

23
12
14
49

4. Let $g(x)=\dfrac{-3x-6}{x^2+2x-3}$. *Find the following:*

a) x-intercepts, if any $x=-2$ (or $(-2,0)$) Let $-3x-6=0$; $3x+6=0$; $3x=-6$; $x=-2$

b) y-intercept, if any $y=2$ or $(0,2)$ Let $x=0$; $g(0)=\dfrac{-3(0)-6}{0^2+2(0)-3}=\dfrac{-6}{-3}=2$

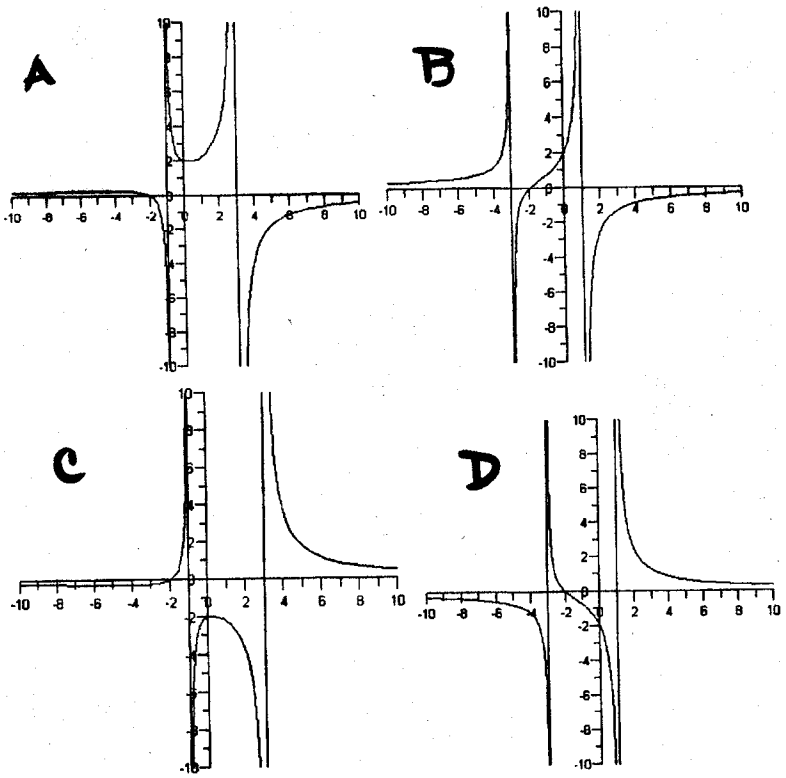
c) Domain $\{x \mid x\neq -3,\ x\neq 1\}$ Let $x^2+2x-3=0=(x+3)(x-1)$ $x=-3$, $x=1$

d) Vertical asymptote(s), if any $x=-3$, $x=1$

e) Horizontal asymptote, if any $y=0$ (degree of denom > degree of numerator

f) Which of the following could be the graph of $g(x)$? B Briefly justify your answer.

A & B match x & y intercepts only B matches VAs.



5. Find the domain of each of the functions below

a) $h(x)=\ln(x)+\ln(x+3)$: $x>0 \cap x>-3 \Rightarrow \boxed{x>0}$

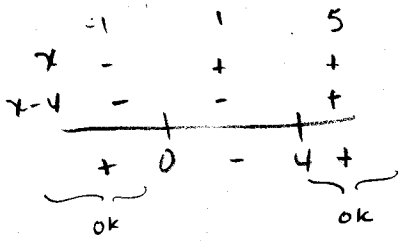
b) $f(x)=x^3e^{5x^4}$ $\mathbb{R}$ (no problems anywhere)

c) $g(x)=\ln(x(x-4))$

let $(x)(x-4)>0$

Solve: $x=0$ $x=4$

$\{x \mid x<0 \text{ or } x>4\}$



6. True or False. Indicate whether each of the following statements is true or false. You must show work justifying your answer in order to receive full credit.

16

13 T/F (+ work)

a) T $\log 50 - \log \frac{1}{2} = 2$

$= \log\left(\frac{50}{1/2}\right) = \log 50 \cdot 2 = \log 100 = \log 10^2 = 2$ ✓

b) F $(\ln 2)\cdot(\ln 50) = \ln(100)$

↳ $\ln 100 = \ln(2\cdot 50) = \ln 2 + \ln 50$

c) T $4^{2\log_4 3} = 9$: $4^{\log_4 3^2} = 4^{\log_4 9} = 9$ ✓

d) F $\ln 6 = \frac{\ln 12}{\ln 2}$

↳ $\ln\left(\frac{12}{2}\right) = \ln 12 - \ln 2 \neq \frac{\ln 12}{\ln 2}$

7. Use the laws of logarithms to rewrite the expression below in a form with no logarithms of a product, quotient or power:

10

$$\ln\left(x^5 e^4 (y^2-2)^4\right)$$

$\ln x^5 + \ln e^4 + \ln(y^2-2)^4 =$

$5\ln x + 4 + 4\ln(y^2-2)$

8. Solve for x. Where necessary, leave your answer in exact form.

4 ea

12

a) $2\ln x = \ln 4 + \ln(x+2)$

4 $\ln x^2 = \ln 4(x+2)$

$x^2 = 4x + 8$

$x^2 - 4x - 8 = 0$

$x = \frac{4 \pm \sqrt{4^2 - 4(1)(-8)}}{2}$

$= \frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$

$x = 2+2\sqrt{3}$, $x = 2-2\sqrt{3}$ Not in domain

b) $\log(x-6) - \log(3) = 1$

$\log\left(\frac{x-6}{3}\right) = 1$

$10^1 = \frac{x-6}{3}$ $30 = x-6$

$\boxed{36 = x}$

c) $\frac{30}{1+e^{-x}} = 5$ $30 = 5(1+e^{-x})$

$6 = 1+e^{-x}$

$5 = e^{-x}$ $\ln 5 = -x$

$x = \boxed{-\ln 5 = -\ln \frac{1}{5}}$

38 +1