Math 105, Test 1
Problems from old tests
Fall 2009

1. If $A = \{x \mid x < 5\}$, $B = \{x \mid -1 < x \le 6\}$ and $C = \{x \mid x > 2\}$, find the indicated sets. You may express your answer in either set or interval notation.
   a) $A \cup B$

   b) $A \cap B$

   c) $A \cup C$

   d) $B \cap C$

2. Factor the following completely:
   a) $8x^3 - 32xy^2$

   b) $x^3 - 2x^2 - 8x$

3. Find all real solutions to the following. Use the technique of completing the square at least once.
   a) $x^2 + 2x - 4 = 0$

   b) $2x^2 + x = 3$

   c) $x^2 - 2x + 5 = 0$

   d) $\dfrac{1}{x+1} - \dfrac{2}{x^2} = 0$

   e) $|x - 3| = 5$

4. Solve the inequalities. Express your answer in set or interval form.
   a) $\dfrac{4}{x} \le -1$

   b) $|x + 4| > 2$

   c) $2x^2 + x \ge 1$

   d) $1 \le 4 - 3x \le 16$

5. Consider the points $P(5,-1)$ and $Q(3,7)$.
   a) What is the slope of the line through the two points?

   b) What is the equation of the line through the two points, in slope-intercept form?

   c) What is the equation of the line **parallel** to the line you found in part b), through the point (0,2)?

   d) What is the equation of the line **perpendicular** to the line you found in part b), through the point (1,1)?

   e) Is the point (-2, 27) on the line you found above? Is the point (1, 16) on the line? Justify your answers.

   f) What is the distance between the two points?

   g) What is the equation of the circle, centered at $P(5,-1)$, passing through $Q(3,7)$?

6. Which of the points $C(-4,3)$ or $D(3,1)$ is closer to the point $E(-1,2)$?

7. Determine whether the equation $x^2 + y^2 + 2x - 4y - 4 = 0$ represents a circle, a point, or has no graph. If the equation is that of a circle, find its radius and center.

8. A geologist uses a probe to measure the temperature T (in ºC) of the soil at various depths below the surface, and finds that at a depth of x centimeters, the temperature is given by the equation $T = 0.08x - 4$.
   a) True or false: The equation that expresses the temperature of the soil is a quadratic equation.
   b) What is the temperature of the soil at a depth of 1 meter (100 centimeters)?
   c) What do the slope, the x-intercept, and the T-intercept of the graph of this equation represent?
9. Find the domain of the functions:
   a) $f(x) = \sqrt{x^2 - 16}$

   b) $g(x) = \dfrac{x}{x^2 - 5x - 6}$

10. Let $g(x) = x^2 + 4$. Evaluate $g(x)$ at the indicated values:

    a) $g(-2)$
    b) $g(1)$
    c) $g(a)$
    d) $g(a + h)$
    e) $g(\frac{1}{x})$

11. Consider the piecewise defined function below:

$$f(x)=\begin{cases} x^2, & if \ \ x>2 \\ x-1, & if \ \ x<2 \\ 3, & if \ \ x=2 \end{cases}$$

Find $f(3),\ f(2),\ f(-1)$. Be sure to show all work.