Math 105 - Chapter 2 - Questions from old tests - Fall 2009

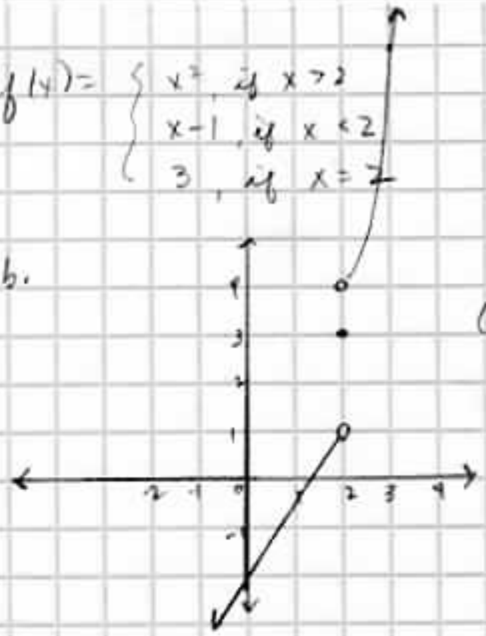
1. $f(x) = \begin{cases} x^2, & \text{if } x > 2 \\ x-1, & \text{if } x < 2 \\ 3, & \text{if } x = 2 \end{cases}$

a) $f(3) = 3^2 = \boxed{9}$

$f(2) = \boxed{3}$

$f(-1) = -1-1 = \boxed{-2}$

b.



(Note: show endpoints of intervals with open circles)

2. $h(x) = x^2 + 4$ Domain: $\mathbb{R}$

a) average rate of change between $x = 1$ & $x = 3$:

$$\frac{3^2+4-(1^2+4)}{3-1} = \frac{9+4-1-4}{2} = \frac{8}{2} = \boxed{4}$$

b) average rate of change between $x = a$ and $x = a+h$.

$$\frac{(a+h)^2+4-(a^2+4)}{a+h-a} = \frac{\cancel{a^2}+2ah+h^2+\cancel{4}-\cancel{a^2}-\cancel{4}}{h} =$$

$$\frac{2ah+h^2}{h} = \frac{\cancel{h}(2a+h)}{\cancel{h}} = \boxed{2a+h}$$

c) $f(x) = h(\sqrt{x}) = (\sqrt{x})^2 + 4 = x+4$ (with restriction below)

$g(x) = \sqrt{h(x)} = \sqrt{x^2+4}$ Does Not simplify

d) Domain of $f(x)$: domain of $\sqrt{x}$: $x \geq 0$ (greatest possible domain)

Domain of $x+4$: $\mathbb{R}$. $\boxed{\therefore \text{Domain of } f(x) = \{x \mid x \geq 0\}}$

e) domain of x^2+4: $\mathbb{R}$. Domain of $\sqrt{x^2+4}$: $x^2+4 \geq 0$ - true for all $x \in \mathbb{R}$,

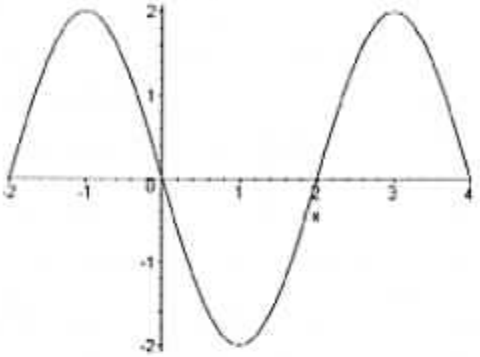
so $\boxed{\text{domain of } g(x) \text{ is } \mathbb{R}.}$

3. $g(x) = \frac{1}{x+1}$. Average rate of change between $x=h$ and $x=0$ is:

$$\frac{g(h)-g(0)}{h-0} = \frac{\frac{1}{h+1} - \frac{1}{0+1}}{h-0} = \frac{\frac{1}{h+1} - 1}{h} = \frac{\frac{1}{h+1} - \frac{h+1}{h+1}}{h} = \frac{\frac{1-h-1}{h+1}}{h} =$$

$$\frac{\frac{-h}{h+1} \cdot \frac{1}{h}}{h \cdot \frac{1}{h}} = \frac{\frac{-h}{h(h+1)}}{1} = \boxed{\frac{-1}{h+1}}$$

4. Consider the graph below:



a) Does it represent the graph of a function? yes Why or why not? Passes vertical line test.
Note: Not a one-to-one function, because it fails horizontal line test.

b) In interval notation, state the domain and range of this graph:
Domain: [-2, 4] Range: [-2, 2]

c) Over what interval(s) is the graph increasing? (-2, -1) and (1, 3)

Questions from book:
a) $f(0) = 3$ is larger than $g(0) \approx 0.5$
b) $g(-3) \approx 2.9$ is larger than $f(-3) \approx -1$
c) $f(x) = g(x)$ at $x = -2$ and at $x = 2$.

5. Page 167, Problem 25, information from a graph. In addition to questions in the book answer:

a) What is the domain of *f*? [-4, 4]

b) What is the range of *f*? [-2, 3]

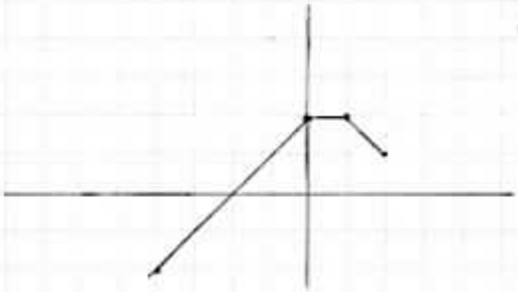
c) What is the domain of *g*? [-4, 3]

d) What is the range of *g*? [0.5 (approx), 4]

#6. problem 20, p.191 (problem 21 answers in book)

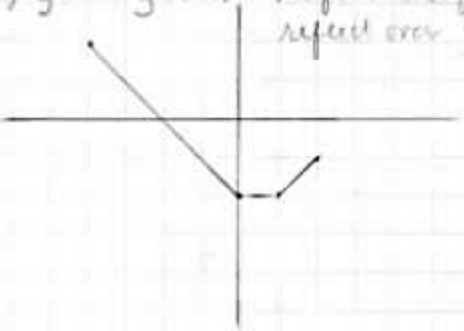
20 a) $y = g(x+1)$: shift 1 to left (horizontal shift)

(Note: New domain: $[-4, 2]$
Range unchanged from $[-2, 2]$



b) $y = -g(x+1)$ shift 1 to left (horizontal)
reflect over x-axis (vertical reflection)

(Note: Domain: $[-4, 2]$
Range: $[-2, 2]$

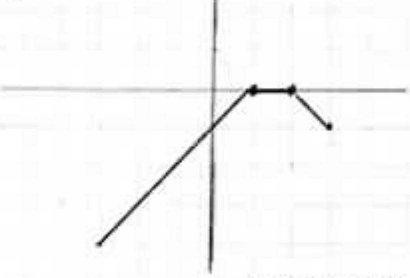


c) $y = g(x-2)$ horizontal shift: 2 to right

(Domain: $[-1, 5]$
Range $[-2, 2]$

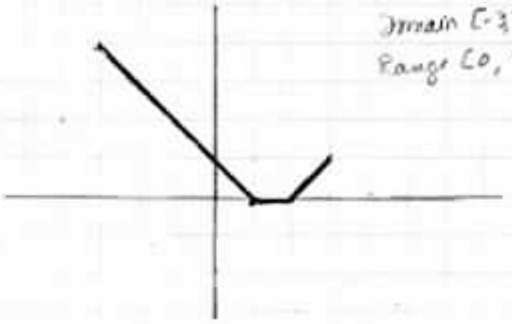
d) $y = g(x) - 2$ vertical shift, down 2

Domain $[-3, 3]$
Range $[-4, 0]$



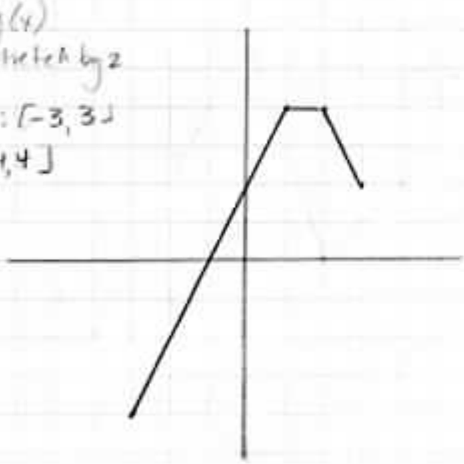
e) $y = -g(x) + 2$ reflect across x-axis, move up 2 } vertical shifts

Domain $[-3, 3]$
Range $[0, 4]$

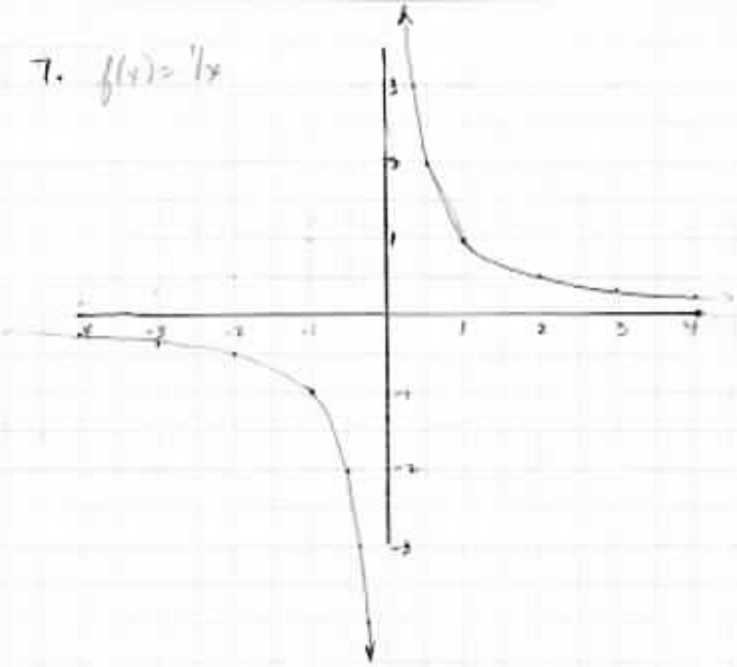


f) $y = 2g(x)$
vertical stretch by 2

(Domain: $[-3, 3]$
Range $[-4, 4]$

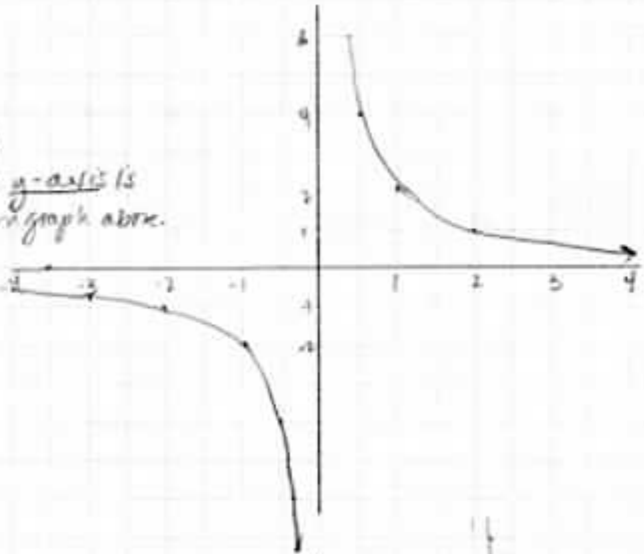


7. $f(x) = 1/x$

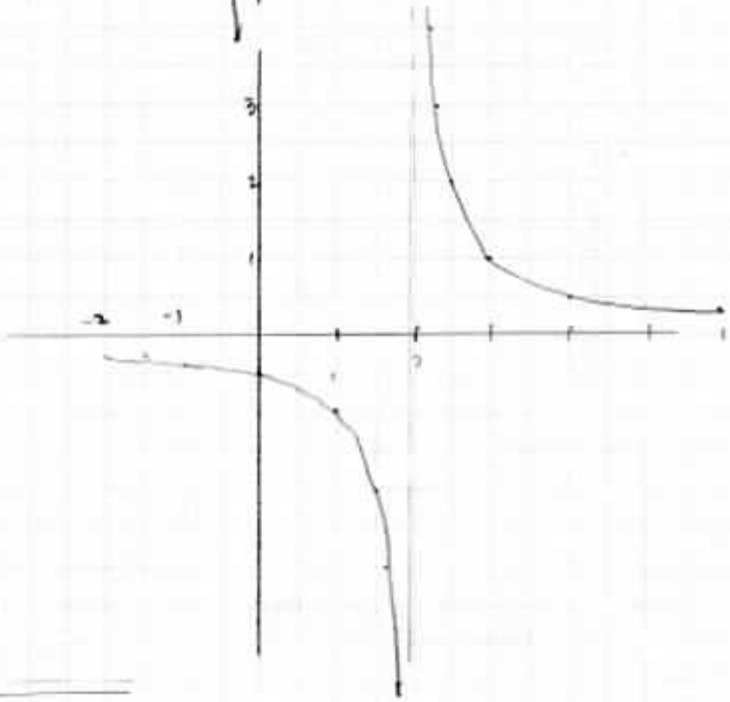


x	$1/x$	x	$1/x$
-4	-1/4	-1/2	-2
-3	-1/3	-1/3	-3
-2	-1/2	-1/4	-4
-1	-1	-1/5	5
1	1	1/2	2
2	1/2	1/3	3
3	1/3	1/4	4
4	1/4	1/5	5

i. $y = \frac{2}{x}$ (vertical stretch by 2)
NOTE: Scale on y-axis is different from graph above.



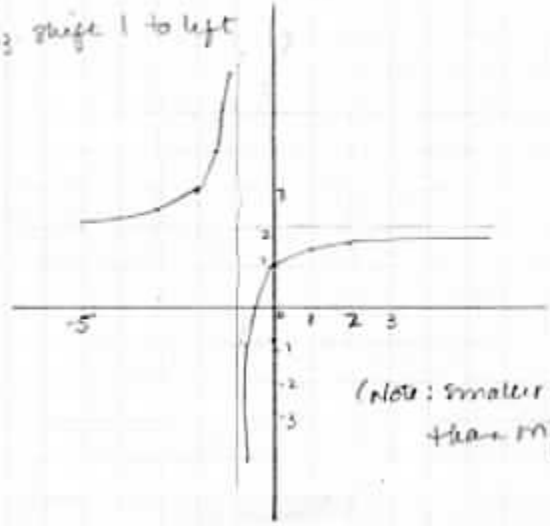
ii $y = \frac{1}{x-2} = f(x-2)$ (shift right by 2 - horiz. shift.)



iii $y = 2 - \frac{1}{x+1}$

shift up by 2 & reflect over x-axis

horiz shift 1 to left



(Note: smaller scale than original.)

8. $f(x) = -2x^2 + 8x - 6$

a) $= -2(x^2 - 4x) - 6$

$= -2(x^2 - 4x + 4 - 4) - 6$ complete the square, but collect everything inside ()'s

$= -2(x-2)^2 + 8 - 6$

$= -2(x-2)^2 + 2$ Standard form

$[a(x-h)^2 + k]$

b) vertex is at $(-h, k) = (2, 2)$

c) x intercepts: let $-2x^2 + 8x - 6 = 0$

$x^2 - 4x + 3 = 0$

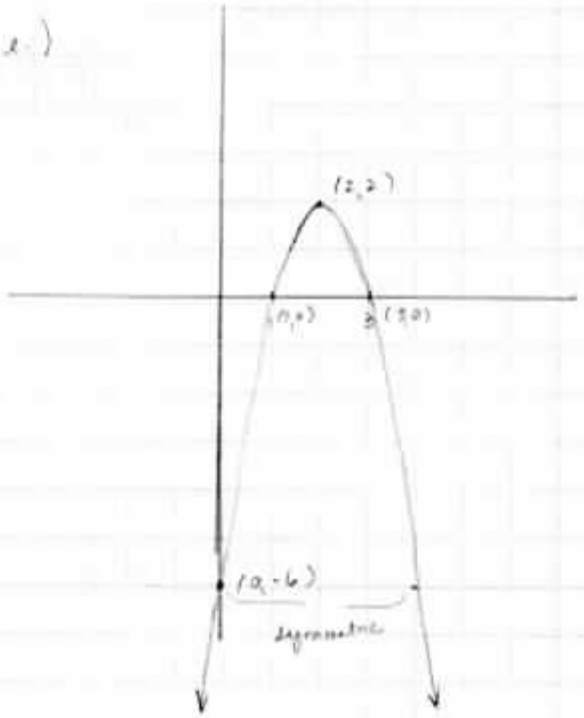
$(x-3)(x-1) = 0$

$x = 3$ and $x = 1$ are x-intercepts

y intercept: $f(0) = -2 \cdot 0^2 + 8 \cdot 0 - 6 = -6$. $(0, -6)$ is y-intercept.

d) $f(x)$ is a parabola opening down. Therefore its maximum is at $(2, 2)$

e.)



9. $f(x) = 1/x$ $g(x) = \sqrt{4+x}$

a) $\dfrac{f(x)}{g(x)} = \dfrac{\frac{1}{x}}{\sqrt{4+x}} = \dfrac{1}{(x)(\sqrt{4+x})}$ Domain: 2 issues: $x \neq 0$, $x > -4$

$\therefore$ Domain is intersection of these 2 sets: $\{x \mid x > -4, x \neq 0\}$

b) $f(x) - g(x) = \dfrac{1}{x} - \sqrt{4+x}$ Domain: $x \neq 0$, $x \geq -4$ or $\{x \mid x \geq -4, x \neq 0\}$

$\uparrow$ may $= 0$.

c) $f(g(x)) = f(\sqrt{4+x}) = \dfrac{1}{\sqrt{4+x}}$ Domain: $x > -4$ (not $= -4$)

d) $f(f(x)) = \dfrac{1}{\frac{1}{x}} = x$ Domain: $\{x \mid x \neq 0\}$ (Must look at each step.)

c) $g(f(x)) = g(1/x) = \sqrt{4 + 1/x}$

Domain: Domain of f: $x \neq 0$

Domain of $g \circ f$: $4 + 1/x \geq 0$

$\frac{4x+1}{x} \geq 0$ (Recall solving inequalities)

Let $4x+1=0$ $x=0$

$x = -1/4$

	-1	$-1/8$	1
$4x+1$	$-$	$+$	$+$
x	$-$	$-$	$+$
	$+$	$-$	$+$

$+ \;\; -1/4 \;\; - \;\; 0 \;\; +$

$\boxed{\therefore \text{Domain of } g(f(x)) = \{x \mid x \leq -1/4 \text{ or } x > 0\} \text{ or } (-\infty, -1/4] \cup (0, +\infty)}$

10. $F(x) = (x+7)^4$ $\boxed{\text{Let } g(x) = x+7,\; f(x) = x^4}$

Then $f \circ g(x) = f(x+7) = (x+7)^4$

11. $f(x) = \frac{2}{x+4}$ a) Domain of f: $\{x \mid x \neq -4\}$

b) $f(x)$ is 1-to-1. Reasons: just a stretch & shift of $f(x) = \frac{1}{x}$, which we have demonstrated is one-to-one. OR:

Let $f(x_1) = f(x_2)$

Then $\frac{2}{x_1+4} = \frac{2}{x_2+4}$; $\frac{x_1+4}{2} = \frac{x_2+4}{2}$;

$x_1 + 4 = x_2 + 4$; $x_1 = x_2$. $\therefore$ f must be 1-to-1.

c) Let $y = \frac{2}{x+4}$. $x+4 = \frac{2}{y}$; $x = f^{-1} = \frac{2}{y} - 4$

d) $f \circ f^{-1} = f\left(\frac{2}{y} - 4\right) = \frac{2}{\frac{2}{y} - 4 + 4} = \frac{2}{\frac{2}{y}} = \cancel{2} \cdot \frac{y}{\cancel{2}} = y$ ✓

$f^{-1} \circ f = f^{-1}\left(\frac{2}{x+4}\right) = \frac{2}{\frac{2}{x+4}} - 4 = \cancel{2} \cdot \frac{x+4}{\cancel{2}} - 4 = x + 4 - 4 = x$ ✓

e) domain of f (= range of f^{-1}) is $\{x \mid x \neq -4\}$

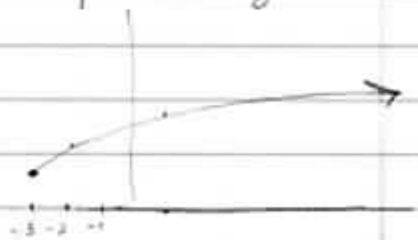
domain of f^{-1} (= range of f) is $\{y \mid y \neq 0\}$.

11. continued

$g(x) = 1 + \sqrt{x+3}$

a) Domain of g: $\{x \mid x \geq -3\}$

b) g is one-to-one because it is a horizontal shift (3 to left) and vertical shift (up 1) of $f = \sqrt{x}$; and we know $f = \sqrt{x}$ passes horizontal line test.



c) let $y = 1 + \sqrt{x+3}$

$y - 1 = \sqrt{x+3}$

$(y-1)^2 = (\sqrt{x+3})^2 = x+3$

$y^2 - 2y + 1 - 3 = x$

$\boxed{y^2 - 2y - 2 = x = g^{-1}}$

d) $g(g^{-1}(y)) = g(y^2 - 2y - 2) = 1 + \sqrt{y^2 - 2y - 2 + 3} = 1 + \sqrt{y^2 - 2y + 1} =$

$1 + \sqrt{(y-1)^2} = 1 + y - 1 = y$ ✓

$g^{-1}(g(x)) = g^{-1}(1 + \sqrt{x+3}) = (1 + \sqrt{x+3})^2 - 2(1 + \sqrt{x+3}) - 2 =$

$1 + 2\sqrt{x+3} + (\sqrt{x+3})^2 - 2 - 2\sqrt{x+3} - 2 =$

$1 + x + 3 - 4 = x$ ✓

e) Domain of g^{-1}: $y \geq 1$ = range of g.

(Note that $y^2 - 2y - 2$ is defined for all real numbers, but it is one-to-one only from the vertex $\rightarrow$ $\frac{-b}{2a} = \frac{-(-2)}{2} = 1$. So $y \geq 1$ is domain of one-to-one portion of the function.)